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ANNUITIES
ON
LIVES:

WITH
Several TABLES, exhibiting at one View,
the VALUES of LIVES, for different
RATES of INTEREST.

FOURTH EDITION,

In which are added,

TABLES for Three and Three and
a Half *per Cent.*

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Fellow of the Royal Societies of *London* and
Berlin.

LONDON:

Printed for A. MILLAR, over-againſt *Catherine-ſtreet,* in the *Strand.* MDCCLII.

TO THE

RIGHT HONOURABLE

GEORGE

Earl of MACCLESFIELD.

MY LORD,

I HAVE had the Honour of dedicating Three Editions of this Work; the Firſt to your Noble Father, the other Two to your Lordſhip; who, in a continual Endeavour to promote Arts and Sciences, eſpecially thoſe called Mathematical: in a conſtant Benevolence to all Mankind, particularly to thoſe who ſtudy the Good of Society; and in a regular Diſcharge of all the important Duties of Life, are truly his Succeſſor.

I can have no Pretence to ſeek, elſewhere, for a Patron to this Fourth Edition, which the Demand I have met with for the Copies, and ſome typographical Errors, heretofore overlooked, have rendered neceſſary; and therefore I again treſpaſs on your Lordſhip's Indulgence in this

Addreſs, well knowing, that your uſual Candour and Goodneſs will excuſe any Imperfections that may ſtill remain in the Performance of

MY LORD,

Your Lordſhip's

moſt Obedient, and

moſt humble Servant,

A. De Moivre.

PREFACE

TO THE

Second EDITION.

DR. *Halley* publiſhed in the *Philoſophical Tranſactions*, N°. 196. an Eſſay concerning the Valuation of Lives; it was partly built upon five Years Obſervation of the Bills of Mortality taken at *Breſlaw*, the Capital of *Sileſia*, and partly on his own Calculation.

Altho' he had thereby confirmed the great Opinion which the World entertain'd of his Skill and Sagacity, yet he was ſenſible, that his Tables and Calculations were ſuſceptible of farther Improvements; of this he expreſs'd his Senſe in the following Words; *Were this Calculus founded on the Experience of a very great Number of Years, it would very well be worth the while, to think of Methods to facilitate the Computation of two, three, or more Lives.*

From whence it appears, that the Table of Obſervations being only the Reſult of a few Years Experience, it was not ſo entirely to be depended upon, as to make it the Foundation of a fix'd and unalterable Valuation of Annuities on Lives; and that even admitting ſuch a Table could be obtain'd, as might be grounded on the Experience of a great Number of Years, ſtill the Method of applying it to the Valuation of ſeveral Lives, would be extremely laborious, conſidering the vaſt Number of Operations, that would be requiſite to combine every Year of each Life with every Year of all the other Lives.

The Subject of Annuties on Lives, had been long neglected by me, partly prevented by other Studies, partly wanting the neceſſary means to treat of it as it deſerved: But two or three Years after the Publication of the firſt Edition of my *Doctrine of Chances*, I took the Subject into conſideration; and conſulting Dr. *Halley*'s Table of Obſervations, I found that the Decrements of Life, for conſiderable Intervals of Time, were in Arithmetic Progreſſion; for Inſtance, out of 646 Perſons of twelve Years of Age, there remain 640 after one Year; 634 after two Years; 628, 622, 616, 610, 604, 598, 592, 586, after 3, 4, 5, 6, 7, 8, 9, 10 Years reſpectively, the common Difference of thoſe Numbers being 6.

Examining afterwards other Caſes, I found that the Decrements of Life for ſeveral Years were ſtill in Arithmetic Progreſſion; which, may be obſerved from the Age of 54, to the Age of 71, where the Difference for 17 Years together, is conſtantly 10.

After having thoroughly examined the Tables of Obſervation, and diſcovered that Property of the Decrements of Life, I was inclined to compoſe a Table of the Values of Annuities on Lives, by keeping cloſe to the Tables of Obſervation; which would have been done with Eaſe, by taking, in the whole Extent of Life, ſeveral Intervals whether equal or unequal: However, before I undertook the Taſk, I tried what would be the Reſult, of ſuppoſing thoſe Decrements uniform from the Age of Twelve; being ſatisfied that the Exceſſes ariſing on one ſide, would be nearly compenſated by the Defects on the other; then comparing my Calculation with that of Dr. *Halley*, I found the Concluſion ſo very little different, that I thought it ſuperfluous to join together ſeveral different Rules, in order to compoſe a ſingle one: I need not take notice, that from the Time of Birth to the Age of Twelve, the Probabilities of Life increaſe, rather than decreaſe, which is a Reaſon of the apparent Irregularity of the Tables in the beginning.

Another thing was neceſſary to my Calculation, which was, to ſuppoſe the Extent of Life confin'd to a certain Period of Time, which I ſuppoſed to be at 86: What induced me to aſſume that Suppoſition was 1ſt, That Dr. *Halley* terminates his Tables of Obſervations at the 84th Year; for altho' out of 1000 Children of one Year of Age, there are twenty, who, according to Dr. *Halley's* Tables, attain to the Age of 84 Years, this Number of 20 is inconſiderable, and would ſtill have been reduced, if the Obſervations had been carried two Years farther. 2d. It appears from the Tables of *Graunt*, who printed the firſt Edition of his Book above 80 Years ago, that out of 100 new-born Children, there remain'd not one after 86 Years; this was deduced from the Obſervations of ſeveral Years, both in the City and the Country, at a Time when the City being leſs populous, there was a greater Facility of coming at the Truth, than at preſet. 3d. I was farther confirm'd in my Hypotheſis, by Tables of Obſervation made in *Switzerland*, about the Beginning of this Century, wherein the Limit of Life is placed at 86: As for what is alledged, that by ſome Obſervations of late Years, it appears, that Life is carried to 90, 95, and even to 100 Years; I am no more moved by it, than by the Examples of *Parr*, or *Jenkins*, the firſt of which lived 152 Years, and the other 167.

To this may be added, that the Age for purchaſing Annuities for Life, ſeldom exceeds 70, at which Term, Dr. *Halley* ends his Tables of the Valuation of Lives.

The greateſt Difficulty that occurred to me in this Speculation, was to invent practical Rules that might eaſily be applied to the Valuation of ſeveral Lives; which, however, was happily overcome, the Rules being ſo eaſy, that by the Help of them, more can be perform'd in a Quarter of an Hour, than by any Method before extant, in a Quarter of a Year.

Since the Publication of my firſt Edition, which was in 1724, I made ſome Improvements to it, as may be ſeen in the Second Edition of my *Doctrine of Chances*; but this Edition of the Annuities has many Advantages over the former, and that in reſpect to the Diſpoſition of the Precepts, the Conciſeneſs of the Rules, the Multiplicity of Problems, and Uſefulneſs of the Tables I have invented.

Before I make an End of this Preface, I think it proper to obſerve, that altho' I have given Rules for finding the Value of Annuities for any Rate of Intereſt, yet I have confined myſelf in my Tables, to the ſeveral Rates of 4, 5 and 6 *per Cent.* which may be interpreted, as if I thought it reaſonable, that when Land ſcarce produces three and a half *per Cent.* and South-Sea Annuities barely that Intereſt, yet the Purchaſer of an Annuity ſhould make 4 *per Cent.* or above; but thoſe Caſes can hardly admit of Compariſon, it being well known, that Land in Fee-ſimple procures to the Proprietor Credit, Honour, Reputation, and other Advantages, in conſideration of which, he is contented with a ſmaller Income. As to the Value of South-Sea Annuities, it has its Foundation on the Punctuality of Payments, and on a Parliamentary Security; but Annuities on Lives, have not the former Security, and ſeldom the latter.

It hath been found neceſſary, however, in this fourth *Edition, to add the Tables for* 3 *and* 3½ per Cent. *Intereſt.*

THE VALUATION OF ANNUITIES ON LIVES.

BEFORE I come to the Solution of Queſtions on Lives, it will be neceſſary to explain the Meaning of ſome Words which I ſhall often have occaſion to mention.

1°. SUPPOSING the *Probabilities of Life* to decreaſe in Arithmetic Progreſſion in ſuch manner, as that ſuppoſing, for Inſtance, 36 Perſons each of the Age of 50, if after one Year expired there remain but 35, after two 34, after three 33, and ſo on; it is very plain that ſuch Lives would neceſſarily be extinct in 36 Years, and that therefore the *Probabilities of living* 1, 2, 3, 4, 5, *&c.* Years from this Age of 50 would fitly be repreſented by the Fractions $\frac{35}{36}$, $\frac{34}{36}$, $\frac{33}{36}$, $\frac{32}{36}$, $\frac{31}{36}$, *&c.* which decreaſe in Arithmetic Progreſſion.

I will not ſay that the Decrements of Life are preciſely in that Proportion; ſtill comparing that Hypotheſis with the Table of Dr. *Halley*, from the Obſervations made at *Breſlaw*, they will be found to be exceedingly approaching.

2°. I call that the *Complement of Life*, which remains from the Age given, to the Time of the Extinction of Life, which will be at 86, according to our Hypotheſis. Thus ſuppoſing an Age of 50, becauſe the Difference between 50 and 86 is 36, I call 36 the *Complement of Life.*

3°. I call that the *Rate of Intereſt* which is properly the Amount of one Pound, put out at Intereſt for one Year; otherwiſe one Pound joined with the Intereſt it produces in one Year: thus ſuppoſing Intereſt at 5 *per Cent.* the Intereſt of 1*l.* would be 0.05, which being joined to the Principal 1, produces 1.05; which is what I call the *Rate of Intereſt.*

PROBLEM I.

Suppoſing the Probabilities of Life to decreaſe in Arithmetic Progreſſion, to find the Value of an Annuity upon a Life of an Age given.

SOLUTION.

LET the Rent or Annuity be ſuppoſed $=1$, the Rate of Intereſt $=r$, the Complement of Life $=n$, the Value of an Annuity certain to continue during n Years $=P$, then will the Value of the Life be $\frac{1-\frac{r}{n}P}{r-1}$, which is thus expreſſed in Words at length;

Take the Value of an Annuity certain for ſo many Years, as are denoted *by* *by the Complement of Life; multiply this Value by the Rate of Intereſt, and divide the Product by the Complement of Life, then let the Quotient be ſubtracted from Unity, and let the Remainder be divided by the Intereſt of* 1l. *then this laſt Quotient will expreſs the Value of an Annuity for the Age given.*

THUS ſuppoſe it were required to find the preſent Value of an Annuity of 1*l.* for an Age of 50, Intereſt being at 5 *per Cent.*

THE Complement of Life being 36, let the Value of an Annuity certain, according to the given Rate of Intereſt, be taken out of the Tables annexed to this Book, this Value will be found to be 16.5468.

LET this Value be multiplyed by the Rate of Intereſt 1.05, the Product will be 17.3741.

LET this Product be divided by the Complement of Life, *viz.* by 36, the Quotient will be 0.4826.

SUBSTRACT this Quotient from Unity, the Remainder will be 0.5174.

LASTLY, divide this Quotient by the Intereſt of 1*l.* *viz.* by 0.05, and the new Quotient will be 10.35; which will expreſs the Value of an Annuity of 1*l.* or how many Years Purchaſe the ſaid Life of 50 is worth.

AND in the ſame manner, if Intereſt of Money was at 6 *per Cent.* an Annuity upon an Age of 50, would be found worth 9.49 Years Purchaſe.

BUT as I have annexed to this Treatiſe the Values of Annuities for an Intereſt of 3, 3 $\frac{1}{2}$, 4, 5, and 6 *per Cent.* it will not be neceſſary to calculate thoſe Caſes, but ſuch only as require a Rate of Intereſt higher or lower, or intermediate; which will ſeldom happen, but in caſe it does, the Rule may eaſily be applied.

PROBLEM II.

The Values of two ſingle Lives being given, to find the Value of an Annuity granted for the Time of their joint continuance.

SOLUTION.

LET *M* be the Value of one Life, *P* the Value of the other, *r* the Rate of Intereſt; then the Value of an Annuity upon the two joint Lives will be $\frac{MP}{M+P-\overline{r-1}\,MP}$, in Words thus;

Multiply together the Values of the two Lives, and reſerve the Product.

Let that Product be again multiplied by the Intereſt of 1l. *and let that new Product be ſubtracted from the Sum of the Values of the Lives, and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will expreſs the Value of the two joint Lives.

THUS, ſuppoſing one Life of 40 Years of Age, the other of 50, and Intereſt at 5 *per Cent.* The Value of the firſt Life will be found in the Tables to be 11.83, the Value of the ſecond 10.35, the Product will be 122.4405, which Product muſt be reſerved.

MULTIPLY this again by the Intereſt of 1*l.* *viz.* by 0.05, and this new Product will be 6.122025

THIS new Product being ſubtracted from the Sum of the Lives which is 22.18, the Remainder will be 16.057975, and this is the ſecond Quantity reſerved.

Now dividing the firſt Quantity reſerved by the ſecond, the Quotient will be 7.62 nearly; and this expreſſes the Values of the two joint Lives.

IF the Lives are equal, the Canon for the Value of the joint Lives will be ſhortened and be reduced to $\frac{M}{2-\overline{r-1}\times M}$, which in words may be thus expreſſed;

Take the Value of one Life, and reſerve that Value.

Multiply this Value by the Intereſt of 1 l. *and then ſubtract the Product from the Number* 2, *and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will expreſs the Value of the two equal joint Lives.

THUS, ſuppoſing each Life to be 45 Years of Age, and Intereſt at 5 *per Cent.*

THE Value of one Life will be found to be 11.14, the firſt Quantity reſerved.

THIS being multiplied by 0.05 the Intereſt of 1*l.* the Product will be 0.557.

THIS Product being ſubtracted from the Number 2, the Remainder will be 1.443, the ſecond Quantity reſerved.

DIVIDE the firſt Quantity reſerved *viz.* 11.14; by the ſecond, *viz.* 1.443, and the Quotient 7.72 will be the Value of the two joint Lives, each of 45 Years of Age.

PROBLEM III.

The Values of three ſingle Lives being given, to find the Value of an Annuity for the Time of their joint continuance.

SOLUTION.

LET M, P, Q be the reſpective Values of the ſingle Lives, then the

Value of the three joint Lives will be $\frac{MPQ}{MP+MQ+PQ-2dMPQ}$, ſuppoſing *d* to repreſent the Intereſt of 1 *l.* in Words thus ;

Multiply the Values of the ſingle Lives together, and reſerve the Product.

Let that Product be multiplied again by the Intereſt of 1 l. *and let the Double of that new Product be ſubtracted from the Sum of the ſeveral Products of the Lives taken two and two, and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will be the Value of the three joint Lives.

THUS, ſuppoſing one Life to be worth 13 Years Purchaſe, the ſecond 14, the third 15, and Intereſt at 4 *per Cent.* the Product of the three Lives will be 2730, which being multiplied by the Intereſt of 1 *l. viz.* by 0.04, the new Product will be 109.20, whereof the double is 218.40: Now the Product of the firſt Life by the ſecond is 182; the Product of the firſt Life by the third is 195; and the Product of the ſecond Life by the third is 210, the Sum of all which is 587; from which ſubtracting the Number 218.40 found above, the Remainder will be 368.60, by which the Product of the three Lives, *viz.* 2730 being divided, the Quotient 7.41 will be the Value of the three joint Lives.

BUT if the three Lives were equal, the general Expreſſion of the Value of the joint Lives will be much ſhorter; for let M repreſent the Value of one Life, d the Intereſt of 1 *l.* then the Value of the three joint Lives will be $\frac{M}{3-2dM}$, in Words thus;

Take the Value of one Life, and reſerve it, multiply this Value by the Intereſt of 1 l. *and double the Product.*

Subtract this double Product from the Number 3, and reſerve the Remainder.

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will be the Value of the three joint Lives.

THUS, ſuppoſing three equal Lives each worth 14 Years Purchaſe, reſerve the Number 14.

MULTIPLY this by 0.04, Intereſt of 1*l.* the Product will be 0.56, which being doubled, will be 1.12.

THIS being ſubtracted from the Number 3, the Remainder will be 1.88, which is the ſecond Quantity to be reſerved.

DIVIDE 14, the firſt Quantity reſerved by the ſecond 1.88, and the Quotient 7.44 will be the Value of the three joint Lives.

FROM the two laſt Examples it appears, that in eſtimating the Values of joint Lives, it would be an Error to ſuppoſe that they might be reduced

Value of the longeſt of the two Lives will be $M+P-\overline{MP}$. In Words thus;

From the Sum of the Values of the ſingle Lives, ſubtract the Value of the joint Lives, and the Remainder will be the Value of the longeſt.

LET us ſuppoſe two Lives, one worth 13 Years Purchaſe, the other 14, and Intereſt at 4 *per Cent.* The Sum of the Values of the Lives is 27, the Value of the two joint Lives by the Rules before given, will be found 9.23. Now, ſubtracting 9.23 from 27, the Remainder 17.77 is the Value of the longeſt of the two Lives.

IF the two Lives are equal, the Operation will be ſomething ſhorter.

BUT it is proper to obſerve in this place, that if ſeveral equal Lives are concerned in an Annuity, I commonly denote one ſingle Life by M', two joint Lives by M'', three joint Lives by M''', and ſo on; ſo that the Rule for an Annuity to be granted till ſuch Time as either of the equal Lives is in being may be expreſſed by $2\,M'-M''$.

PROBLEM V.

The Values of three ſingle Lives being given, to find the Value of an Annuity upon the longeſt of them.

SOLUTION.

Let M, P, Q, be the Values of the ſingle Lives, $\overline{MP}$, $\overline{MQ}$, $\overline{PQ}$, the Values of all the joint Lives combined two and two, $\overline{MPQ}$ the Value of the three joint Lives, then the Value of an Annuity upon the longeſt of them is $M+P+Q-\overline{MP}-\overline{MQ}-\overline{PQ}+\overline{NPQ}$, in Words thus;

Take the Sum of the three ſingle Lives, from which Sum ſubtract the Sum of all the joint Lives combined two and two, then to the Remainder add the Value of the three joint Lives, and the Reſult will be the Value of the longeſt of the three Lives.

THUS ſuppoſing the ſingle Lives to be 13, 14, and 15 Years Purchaſe, the Sum of the Values will be 42; the Values of the firſt and ſecond joint Lives is 9.24, of the firſt and third 9.65, of the ſecond and third 10.18, the Sum of all which is 29.06, which being ſubtracted from the Sum of the Lives found before, *viz.* 42, the Remainder will be 12.94, to which adding the Value of the three joint Lives 7.41, the Sum 20.35 will be the Value of the longeſt of the three joint Lives.

BUT if the three Lives are equal, the Rule for the Value of the Life that remains laſt is $3M' - 3M'' + M'''$.

Of REVERSIONS.

PROBLEM VI.

Suppoſe A *is in Poſſeſſion of an Annuity, and that* B *after the Deceaſe of* A *is to have the Annuity for him, and his Heirs for ever, to find the preſent Value of the Reverſion.*

SOLUTION.

LET *M* be the Value of the Life in Poſſeſſion, *r* the Rate of Intereſt, then the preſent Value will be $\frac{1}{r-1}-M$, that is, *from the Value of the Perpetuity, ſubtract the Value of the Life in Poſſeſſion, and the Remainder will be the Value of the Reverſion.*

THUS, Suppoſing that *A* is 50 Years of Age, an Annuity upon his Life, Intereſt at 5 *per Cent.* would be 8.39, which being ſubtracted from the Perpetuity 20, the Remainder will be 11.61, which is the Preſent Value of the Expectation of *B*.

IN the ſame manner, ſuppoſing that *C* were to have an Annuity for him and his Heirs for ever, after the Lives of *A* and *B*, then from the Perpetuity ſubtracting the Value of the longeſt of the two Lives of *A* and *B*, the Remainder will expreſs the Value of *C*'s Expectation.

THUS, Suppoſing the Ages of *A* and *B* be 40 and 50, the Value of an Annuity upon the longeſt of theſe two

Lives would be found by the 4th Problem, to be 14.56; and this being ſubtracted from the Perpetuity 20, the Remainder is 5.44, which is the Value of *C*'s Expectation, and the Rule will be the ſame in any other Caſe that may be propoſed.

PROBLEM VII.

Suppoſing that A *is in Poſſeſſion of an Annuity for his Life, and that* B *after the Life of* A, *ſhould have an Annuity for his Life only; to find the Value of the Life of* B *after the Life of* A.

THIS Caſe ought carefully to be diſtinguiſhed from the Caſe of the 6th Problem; for in that Problem, altho' the Expectant *B* ſhould die before *A*, ſtill the Heirs of *B* have the Reverſion; but in the Caſe of the preſent Problem, if *B* dies before *A*, the Heirs of *B* have no Expectation.

SOLUTION.

LET *M* be the Value of the Life of the preſent Poſſeſſor, *P* the Value of the Life of the Expectant, then the Value of his Expectation is $P - \overline{MP}$. In Words thus;

From the preſent Value of the Life of B, *ſubtract the preſent Value of the joint Lives of* B *and* A, *and the Remainder will be the Value of* B's *Expectation.*

THE Reaſon of which Operation is very plain, for if *B* were now to begin to receive the Annuity, it would be worth to him the Sum *P* in preſent Value; but as he is to receive nothing during the joint Lives of himſelf and *A*, the preſent Value of their two joint Lives ought to be ſubtracted from the Value of his own Life.

PROBLEM VIII

To find the Value of one Life after two.

Thus, Suppoſe A *in Poſſeſſion of an Annuity for his Life, that* B *is to have his Life in it after* A, *and that* C *is likewiſe to have his Life in it after* B, *but ſo that* B *dying before* A, C *ſucceeds* A *immediately; to find the Value of* C's *Expectation.*

So-

SOLUTION.

LET *M*, *P*, *Q*, be the reſpective Values of the Lives of *A*, *B*, *C*, then the Value of *C*'s Expectation is $\begin{array}{l} Q-\overline{MQ}+\overline{MPQ}, \\ \quad -\overline{PQ} \end{array}$ which in Words is thus expreſſed;

From the preſent Value of the Life of C, *ſubtract the Sum of the joint Lives of himſelf and* A, *and of himſelf and* B, *and to the Remainder add the Sum of the three joint Lives, and the Reſult of theſe Operations will expreſs the preſent Value of the Expectation of* C.

PROBLEM IX.

If A, B, C *agree among themſelves to buy an Annuity to be by them equally divided, whilſt they live together, then after the Deceaſe of one of them, to be equally divided between the two Survivors, then to belong entirely to the laſt Survivor for his Life; to find what each*

each of them ought to contribute towards the Purchaſe.

SOLUTION.

LET M, P, Q be the reſpective Values of the Lives of A, B, C, then what A is to contribute, is

$$M - \tfrac{1}{2}\overline{MP} + \tfrac{1}{3}\overline{MPQ} - \tfrac{1}{2}\overline{MQ}$$

What B is to contribute, is

$$P - \tfrac{1}{2}\overline{PM} + \tfrac{1}{3}\overline{MPQ} - \tfrac{1}{2}\overline{PQ}$$

What C is to contribute, is

$$Q - \tfrac{1}{2}\overline{QM} + \tfrac{1}{3}\overline{MPQ} - \tfrac{1}{2}\overline{QP}$$

In Words thus; *From the Value of the Life of* A, *ſubtract the half Sum of the Values of the joint Lives of himſelf and* B, *and of himſelf and* C, *and to the Remainder add* $\frac{1}{3}$ *of the Value of the three joint Lives, and the Sum will be* *what* A *is to contribute towards the Purchaſe.*

IN like manner, from the Value of *B*'s Life ſubtract the half Sum of the Values of the joint Lives of himſelf and *A*, and of himſelf and *C*, and to the Remainder add $\frac{1}{3}$ of the Value of the three joint Lives, and the Sum will be what *B* is to contribute.

AND again, from the Value of the Life of *C*, ſubtract the half Sum of the Values of himſelf and *A*, and of himſelf and *B*; then to the Remainder add $\frac{1}{3}$ of the Values of the three joint Lives, and this laſt Operation will ſhew what *C* is to contribute.

PROBLEM X.

Suppoſing three equal Lives of any Age given, for Inſtance 30, *and that upon the Failing of any one of them, that Life ſhall be immediately replaced, and I then receive a Sum agreed upon, and that no Perpetuity for me and my Heirs; what is the preſent Value*

Value of that Expectation, and at what Intervals of Time, one with another, may I expect to receive the ſaid Sum?

SOLUTION.

IMAGINE that there is an Annuity of 1*l.* to be received as long as the three Lives are in being, and that the Preſent Value is M'''. which Symbol we make uſe of to repreſent the Preſent Value of an Annuity upon three equal joint Lives; now, ſince each Life is ſuppoſed to be 30 Years of Age, and that the Rate of Intereſt is 5 *per Cent.* we ſhall find, by following the Directions of the 3d Problem, that the Preſent Value of the three joint Lives is $7.64 = M'''$; this being fixed, the Preſent Value of all the Payments to be made to Eternity at equal Intervals of Time, will be $\frac{1-dM''}{dM''} \times ſ$, where the Quantity d ſignifies the Intereſt of 1*l.* In Words thus;

Multiply the Preſent Value of the three joint Lives, viz. 7.64, *by the Intereſt* *of* 1l. *which in this Caſe is* 0.05, *and that Product, which is* 0.382, *muſt be reſerved.*

Subtract this Quantity from Unity, and the Remainder, viz. 0.618 *being divided by the Quantity reſerved, the Quotient will be* 1.62, *and this being multiplied by the Sum* f, *which we may ſuppoſe* 100 l. *the Product will be* 162 l. *and this is the preſent Value of all the Payments that will be made to Eternity, at equal Intervals of Time upon the failing of a Life, which is to be immediately replaced.*

As for the Intervals of Time after which thoſe Replacements will be made, they may be found thus;

LOOK in the ſecond of our Tables for the Number 7.64, which is the Value of the three joint Lives, and over againſt it will be found the Number anſwering, which is between 9 and 10; and ſo it may be ſaid that the Replacements will be made at every Interval of about 9 or 10 Years.

BUT that Interval may be determined a little more accurately, by help of a Table of Logarithms, by taking the Logarithm of the Quantity $\frac{1}{1 - dM'''}$ and dividing it by the Logarithm of r.

THE Logarithm of $\frac{1}{1 - dM''}$ is 0.2090115; the Logarithm of r is 0.0211893; and the firſt being divided by the ſecond, the Quotient is 9.86, which ſhews that the Replacements will be made at Intervals a little more than $9\frac{3}{4}$ Years.

PROBLEM XI.

Suppoſing, as before, three equal Lives of 30, *and that the Lives are not to be renewed, till after the failing of any two of them, and that a Sum* p *is then to be received, and that perpetually, after the failing of two Lives, what is the preſent Value of that Expectation?*

SOLUTION.

MAKE $3M'' - 2M''' = A$, let the Intereſt of 1 *l.* be $= d$, then the preſent Value of that Expectation will be $\frac{1-dA}{dA} \times p$.

BUT to know the Intervals of Time after which the Lives will be filled up, take the Logarithm of the Quantity $\frac{1}{1-dA}$, and divide it by the Logarithm of r.

THE Value of a ſingle Life of 30, Intereſt at 5 *per Cent.* is found in our Tables to be 13 Years Purchaſe, the Value M'' of two joint Lives by Problem II. is 9.63; and the Value M''' of three joint Lives by Problem III. is 7.64; then $3M'' - 2M'''$, or the Difference between the Triple of two joint Lives, and the Double of three joint Lives will be $13.59 = A$, then $\frac{1-dA}{dA} \times p$ will be found to be $0.473\,p$, and the Intervals of Time will be 23.32, that is, nearly $23\frac{1}{3}$ Years.

PROBLEM XII.

Suppoſing ſtill the Lives to be 30, and that they are not to be renewed till after the Extinction of all three, and

and that a Sum q *is then to be received, and that perpetually after every Renewal, what is the preſent Value of that Expectation?*

SOLUTION.

Make $3M' - 3M'' + M''' = B$, then the preſent Value of that Expectation will be $\frac{1-dB}{dB} \times q$; here B will be found to be 17.76, and conſequently $\frac{1-dB}{dB} \times q$ will be $= 0.121 \times q$.

AND the Intervals of Time will be the Logarithm of the Quantity $\frac{1}{1-dB}$ divided by the Logarithm of r, which in this Caſe would be 44 87, that is, nearly 45 Years.

COROLLARY.

HENCE it will be eaſy for the Proprietor of the Lives, to find which is moſt advantageous to him, to fill up a Life as ſoon as it is vacant, or not to fill up before the Vacancy of two, or to let them all drop before the Renewal.

REMARK.

It is not to be imagined that if Intereſt of Money was higher or lower than 5 *per Cent.* the Intervals of Time after which the Renewals are made, would be the ſame as they are now, for it will be found, that as Intereſt is higher, the Intervals will be ſhorter; and as it is lower, ſo the Intervals will be longer; yet one might make it an Objection to our Rules, that the length of Life would thereby ſeem to depend upon the Rate of Intereſt. To anſwer this Difficulty, it muſt be obſerved, that the calculating of Time imports no more, than that conſidering the Circumſtances of the Purchaſer and the Proprietor of the Lives, in reſpect to the Rate of Intereſt agreed upon, and the Sum to be given upon the Renewal of a Life, or Lives, the Proprietor makes the ſame Advantage of his Money, as if he had agreed with the Purchaſer, that he ſhould pay him a certain Sum of Money at equal Intervals of Time, for redeeming the Riſque which he the Purchaſer runs of paying that Sum when the Life or Lives drop: but the real Intervals of Time will be ſhewn afterwards.

ALTHO' it ſeldom happens that in Contracts about Lives, any more than three are concern'd, yet I hope it will not be diſpleaſing to our Readers to have this Speculation carry'd a little farther.

BUT as general Rules are beſt inculcated by particular Examples, I ſhall take the Caſe of five Lives, and expreſs the ſeveral Circumſtances of them in ſuch manner, as that they may be a ſure Guide to all other Caſes of the ſame kind, let the Number of Lives be what it will; let therefore the following Expreſſions be written,

$$\begin{array}{r} M''''' \\ 5M''''-4M''''' \\ 10M'''-15M''''+6M''''' \\ 10M''-20M'''+15M''''-4M''''' \\ 5M'-10M''+10M'''-5M''''+1M''''' \end{array}$$

THE firſt Term M''''' repreſents properly the preſent Value of an Annuity upon five equal joint Lives, but from thence may be deduced the Time of their joint continuance, or the Time in which it may be expected that one of them will fail, it being as I have ſaid before, the Logarithm of $\frac{1}{1-dM'''''}$ divided by the Logarithm of r: however, for ſhortneſs ſake, I call for the preſent that Expreſſion the Time.

THE two next Terms, $5M''''-4M'''''$, repreſent the Time in which two of the Lives will fail.

THE three next Terms, $10M'''-15M''''+6M'''''$, repreſent the Time in which three out of the five Lives will fail.

THE four next, $10M''-20M'''+15M''''-4M'''''$, repreſent the Time in which four out of the five Lives will fail.

THE five next, $5M'-10M''+10M'''-5M''''+1M'''''$, repreſent the

Time in which all the five Lives will be extinct.

Now the Law of the Generation of the Co-efficients is thus.

1°. Take all the Terms which are affected with the Mark M''''', beginning from the uppermoſt, with the Co-efficients $1-4+6-4+1$, which are the Terms of the Binomial $1-1$, rais'd to the fourth Power, which is leſs by one than the Number of Lives concerned.

2°. Take the Terms which are affected with the Mark M'''', and prefix to them in order, the product of the Number 5 by the Co-efficients $1-3+3-1$, which are the Terms of the Binomial $1-1$ raiſed to its Cube, that is, to a Power leſs by two than the Number of Lives concerned.

3°. Take all the Terms which are affected with the Mark M''', and prefix to them in order, the Product of the Number 10, multiplied by the Co-efficients $1-2+1$, which are the Terms of the Binomial $1-1$ raiſed to its Cube, that is, to a Power leſs by

three

three than the Number of Lives concerned.

4°. TAKE all the Terms which are affected with the Mark M'', and prefix to them the Product of the Number 10, multiplied by the Terms of the Binomial 1—1, raiſed to the Power whoſe Index is 1, that is to a Power leſs by four than the Number of Lives concerned.

5°. TAKE all the Terms which are affected with the Mark M', and prefix to them the Product of the Number 5, multiplied by the Binomial 1—1, raiſed to a Power leſs by 5 than the Number of Lives concerned; which in this Caſe happening to be nothing, or 0, degenerates barely into Unity.

AS for the Multiplicators, conceiving that the Multiplicator of the firſt Term M'''' is 1, all the Multiplicators will be 1, 5, 10, 10, 5, which are all, except the laſt, the Co-efficients of the Binomial 1+1, raiſed to its fifth Power, that is, to a Power equalling the Number of all the Lives.

N.B. THE Exception here given, does not fall upon the Number 5, but upon the laſt Term of the fifth Power, $1+5+10+10+5+1$, which laſt 1 is rejected.

Of SUCCESSIVE LIVES.

PROBLEM XI.

If A *enjoys an Annuity for his Life, and at his Deceaſe has the Nomination of a Succeſſor* B, *who is alſo to enjoy the Annuity for his Life, to find the preſent Value of the two ſucceſſive Lives.*

SOLUTION.

LET the Values of the Lives be M and P; let d be the Intereſt of 1 *l.* then the Value of the two ſucceſſive Lives will be $M+P-dMP$.

BUT if the Succeſſor B was himſelf to have the Nomination of a Life Q; then the Value of the three ſucceſſive Lives would be $M+P+Q-d\times\overline{MP+MQ+PQ}+dd\times MPQ$.

BUT before I proceed, it is proper to obſerve that the Expreſſions MP, MQ, PQ, and MPQ, ſignify barely Products, which is conformable to the uſual Algebraic Notation; this I take notice of, for fear thoſe Expreſſions ſhould be confounded with others that I have made uſe of before, *viz.* $\overline{MP}$, $\overline{MQ}$, $\overline{PQ}$, and $\overline{MPQ}$, which denoted joint Lives.

BUT to compriſe under one general Rule all the poſſible Caſes that may happen about any Number of ſucceſſive Lives, it will be proper to expreſs it in Words at length, thus;

From the Sum of all the Lives, ſubtract the Sum of the Products of all the Lives combined two and two, which Sum of Products before they are ſubtracted, ought to be multiplied by the Intereſt of 1 l.

To this add the Sum of the Products of all the Lives taken three and three, but multiplied again by the Square of the Intereſt of 1 l.

From this ſubtract the Sum of the Products of all the Lives taken four and four, but multiplied again by the Cube of the Intereſt of 1 l. *and ſo on by alternate Additions and Subtractions, ſtill obſerving that if there was occaſion to take the Lives five and five, ſix and ſix,* &c. *the Intereſt of* 1 l. *ought to be raiſed to the* 4*th Power, and to the* 5*th, and ſo on.*

BUT all thoſe Operations would be very much contracted, if the Lives to be nominated were always of the ſame Age, for Inſtance 30: for ſuppoſe *M* to be the Value of an Annuity on an Age of 30, and *d* to be the Intereſt of 1 *l.* then the preſent Value of al the ſucceſſive Lives, of which the Number is *n*, would be $\frac{1-\overline{1-dM}^{n}}{d}$. In Words thus;

Multiply the Value of one Life by the Intereſt of 1 l. *let the Product be ſubtracted from Unity, and let the Remainder be raiſed to that Power which anſwers to the Number of Lives; then* *this Power being again ſubtracted from Unity, let the Remainder be divided by the Intereſt of* 1 l. *and the Quotient will be the preſent Value of all the ſucceſſive equal Lives.*

And again, if the Number of thoſe Lives were infinite, the Sum would barely be $\frac{1}{d}$.

PROBLEM XII.

Of a Perpetual Advowſon.

1°. I ſuppoſe that at the Time of the Demiſe of the Incumbent, the Patron would receive the Sum s, for alienating his Right of the next Preſentation, if the Law did not forbid the Alienation in that Circumſtance of Time.

2°. I ſuppoſe that when this Right is transferr'd, the Age of the Incumbent is ſuch, that an Annuity upon his Life would be worth M Years Purchaſe, when the Intereſt of 1 *l.* is d.

THIS being ſuppoſed, the Right of the next Preſentation is worth $\overline{1-dM}\times ſ$, and the Right of Patronage, or perpetual Recurrency of the like Circumſtances to Eternity, would be worth $\frac{1-dM}{dM}\times ſ$. In Words thus;

Take the preſent Value of the Life of the Incumbent, and multiply it by the Intereſt of 1 l. *and reſerve the Product.*

Subtract this Product from Unity, and let the Remainder be multiplied by the Sum expected ſ, *and the new Product will ſhew the Right of the next Preſentation; let alſo this be reſerved.*

Then divide the ſecond Quantity reſerved by the firſt, and the Quotient will ſhew the preſent Value of the Right of Patronage, or perpetual Recurrency.

THUS, ſuppoſing the Life of the Incumbent worth 8 Years Purchaſe, the Rate of Intereſt 5 *per Cent.* and the Sum *ſ* to be 100 *l.* the Right of the next Preſentation would be worth

60 *l.* and the Right of perpetual Recurrency 150 *l.*

PROBLEM XIII.

Of a Copy-hold.

Suppoſing that every Copy-hold Tenant pays to the Lord of the Manor a certain Fine on Admittance, and that every Succeſſor does the like; to find the Value of the Copy-hold computed from the Time of a Fine being paid, independently from the Fine that may be given on Alienation.

SOLUTION.

I ſuppoſe that the Value of the Life of the preſent Tenant, and the Life of every future Succeſſor when he comes to Poſſeſſion is the ſame; this being admitted, let M be the Value of a Life, d the Intereſt of 1 *l.* and f the Fine to be paid, then the preſent Value of the Copy-hold will be $\frac{1-dM}{dM} \times f$: and this Expreſſion being exactly the ſame as that whereby the Right of Patronage has been determined, needs no Explanation in Words.

ONLY it is neceſſary to obſerve, that the Sum $\int$ paid in Hand being added to this, will make the Canon ſhorter, and will be reduced to $\frac{\int}{dM}$, which may be expreſſed thus in Words.

Divide the Fine by the Product of the Life, multiplied by the Intereſt of 1 l.

THUS, if the Life of a Tenant is worth 12 Years Purchaſe, and the Fine to be paid on Admittance 56*l.* and alſo the Rate of Intereſt 5 *per Cent.* then the preſent Value of the Copyhold is 93 $\frac{1}{3}$*l.*

PROBLEM XIV.

A *borrows a certain Sum of Money, and gives Security that it ſhall be repaid at his Deceaſe with the Intereſts, to fix the Sum which is then to be paid.*

SOLUTION.

LET the Sum borrowed be *ſ*, the Life of the Borrower *M* Years purchaſe, *d* the Intereſt of 1 *l.* then the Sum to be paid at *A*'s Deceaſe will be $\frac{ſ}{1-dM}$; thus, ſuppoſing $ſ=800$, $M=11.83$, $d=0.05$, then $\frac{ſ}{1-dM}$ would be found $=1958$ *l:* in the ſame manner, if the Sum to be paid at *A*'s Deceaſe, was to be an Equivalent for his Life, unpaid at the Time of the Purchaſe, that Sum would be $\frac{M}{1-dM}=$ 2895 *l.* Suppoſing the Annuity received to be 100 *l.* as alſo the Life of *A* 11.83 Years Purchaſe.

PROBLEM XV.

A *borrows a Sum* ſ, *payable at his Deceaſe, but with this Condition, that if he dies before* B, *then the whole Sum is to be loſt to the Lender*; *to find what* A *ought to pay at his Deceaſe in caſe he ſurvives* B.

SOLUTION.

LET us ſuppoſe, as before, that *A* is 40 Years of Age, that the Sum borrowed is 800 *l.* and that Intereſt of Money is 5 *per Cent.* Farther, let it be ſuppoſed that *B* is 70 Years of Age, then, 1°. determine what *A* ſhould pay at his Deceaſe, if the Life of *B* was not concerned; by the Solution of the preceding Problem, we find the Sum to be 1958 *l.* But we ought to conſider that the Lender having a Chance to loſe his Money, there ought to be a Compenſation for the Riſque he runs, which is founded on the poſſibility of a Man of ſeventy outliving a Man of forty. Now, by the Rules to be delivered in the next Problem, we ſhall find that the Probability of that Contingency is meaſured by the Fraction $\frac{4}{23}$, and therefore the Probability of the youngeſt Life's ſurviving the oldeſt is $\frac{19}{23}$. Now this being the Meaſure of the Probability which the Lender has of being repaid, the Sum 1958 ought to be increaſed in the proportion of 23 to 19, which will make it to be 2370 *l.* nearly.

Of the Probabilities of Survivorſhip.

PROBLEM XVI.

Any Number of Lives being given, to find their Probability of Survivorſhip.

SOLUTION.

LET *A*, *B*, *C*, *D*, *&c.* be the Lives, whereof *A* is ſuppoſed to be the youngeſt, *B* the next to it, *C* the next, *&c.* and ſo the laſt the oldeſt.

LET *n*, *p*, *q*, *s*, *t*, *&c.* be the reſpective Intervals intercepted between the Ages of thoſe Lives, and the Extremity of old Age ſuppoſed at 86; then the Probabilities of any one of thoſe Lives ſurviving all the reſt, will be

for *A*	$1-\frac{p}{2n}-\frac{qq}{6np}-\frac{s^3}{12npq}-\frac{t^4}{20npqs}$
B	$+\frac{p}{2n}-\frac{qq}{6np}-\frac{s^3}{12npq}-\frac{t^4}{20npqs}$
C	$+\frac{qq}{3np}-\frac{s^3}{12npq}-\frac{t^4}{20npqs}$
D	$+\frac{s^3}{4npq}-\frac{t^4}{20npqs}$
E	$+\frac{t^4}{5npqs}$
&c.	

Here ſome few things may be obſerved.

1°. THAT the Probability of the youngeſt Life ſurviving all the reſt, always begins with Unity, and that it is expreſſed by ſo many Terms as there are Lives concerned.

2°. THAT the Probabilities of the other Lives ſurviving all the reſt, are always expreſſed each by one Term leſs than the preceding.

3°. THAT each firſt Term of thoſe whereby each Probability is expreſſed, is always the Sum of all the other Terms ſtanding above it.

4°.

4°. THAT the Numbers 2, 6, 12, 20, 30, *&c.* made uſe of in the Denominators of the Fractions are generated by the Multiplication of the following Number, 1 × 2, 2 × 3, 3 × 4, 4 × 5, *&c.* It would take up too much room to explain this general Rule in Words at length, for which Reaſon I ſhall content my ſelf with explaining only the Caſes of two and three Lives, which are the moſt neceſſary.

And, Firſt, *if there be two Lives of a given Age, ſuch as* 40 *and* 70, *take their Complements of Life, which as I have explained before, are the Differences between* 86 *and the reſpective Ages, thoſe Complements therefore are* 46 *and* 16.

Divide the ſhorteſt Complement by the Double of the Longeſt, and the Quotient will expreſs the Probability of the oldeſt Life ſurviving the youngeſt.

THUS in the preſent Caſe, the ſhorteſt Complement being 16, and the double of the longeſt being 92, I divide 16 by 92, and the Quotient $\frac{16}{92}$ or $\frac{4}{23}$ will expreſs the Probability required.

SUBTRACT this Fraction from Unity, and the Remainder $\frac{19}{23}$ will expreſs the Probability of the youngeſt Life ſurviving the oldeſt.

So that the Odds of the youngeſt Life ſurviving the oldeſt, are 19 to 4.

THE Caſe of three Lives is thus: Suppoſe there are three Lives of a given Age, ſuch as 40, 45, and 60; take their reſpective Complements of Life, which are 46, 41, 26, then divide the Square of the ſhorteſt Complement by 3 times the Product of the other two, and the Quotient will expreſs the Probability of the oldeſt Life ſurviving the other two.

DIVIDE the middlemoſt Complement by the Double of the greateſt, and from the Quotient ſubtract the Square of the leaſt divided by 6 Times the Product of the other two, and the Remainder will expreſs the Probability of the middlemoſt Life ſurviving the other two.

SUBTRACT the Sum of the two foregoing Probabilities from Unity, and the Remainder will expreſs the Probability of the youngeſt Life ſurviving the other two.

THUS in the Caſe propoſed, the Probability of the oldeſt Life ſurviving the other two, will be found $\frac{676}{5658}=\frac{3}{25}$ nearly.

THE Probability of the middlemoſt Life ſurviving the other two will be $\frac{4367}{11316}=\frac{5}{13}$ nearly.

THE Probability of the youngeſt Life ſurviving the other two will be $\frac{3}{5}$ nearly.

PROBLEM XVIII.

Any Number of Lives being given, to find the Probability of the Order of their Survivorſhip.

SOLUTION.

SUPPOSE the three Lives to be thoſe of *A*, *B*, *C*, and that it be required to aſſign the Probability of Survivorſhip as limited to the Order in which they are written, ſo that *A* ſhall both ſurvive *B* and *C*, and *B* alſo ſurvive *C*. This being ſuppoſed, let *n*, *p*, *q*, repreſent the reſpective Complements of Life, of the youngeſt, middlemoſt, and oldeſt, then the Probabilities of the ſix different Orders that there are in three things, will be as follows;

A, *B*, *C*	$1 - \frac{p}{2n} - \frac{q}{2p} + \frac{qq}{6nq}$
A, *C*, *B*	$\frac{q}{2p} - \frac{qq}{3np}$
B, *A*, *C*	$\frac{p}{2n} - \frac{q}{2n} + \frac{qq}{6np}$
B, *C*, *A*	$\frac{q}{2n} - \frac{qq}{3np}$
C, *A*, *B*	$\frac{qq}{6np}$
C, *B*, *A*	$\frac{qq}{6nq}$

In Words thus;

1°. *Divide the middlemoſt Complement by the double of the greateſt, and let the Quotient be ſubtracted from Unity.*

2°. *From that Remainder ſubtract again the Quotient of the ſhorteſt Complement divided by the Double of the Middlemoſt.*

3°. *To that new Remainder add the Quotient ariſing from the Square of the ſhorteſt Complement divided by ſix times the Product of the greateſt and middlemoſt multiplied together, and this laſt Sum will expreſs the Probability of the firſt Order.*

THE Probability of the Second will be found thus;

1°. *Divide the ſhorteſt Complement by the double of the middlemoſt, and reſerve the Quotient.*

2°. *Divide the Square of the ſhorteſt by three times the Product of the longeſt Complement, multiplied by the Middlemoſt, and reſerve the new Quotient.*

3°. *Let the ſecond Quotient be ſubtracted from the firſt, and the Remainder will expreſs the Probability of the happening of the ſecond Order.*

THE Probability of the third Order will be found as follows.

1°. *Divide the middlemoſt Complement by the Double of the Greateſt, and reſerve the Quotient.*

2°. *Divide the ſhorteſt Complement by the double of the longeſt, and reſerve the Quotient.*

3°. *Divide the Square of the ſhorteſt Complement by ſix times the Product of the longeſt and middlemoſt multiplied together, and reſerve the Quotient.*

4°. *From the firſt Quotient reſerved, ſubtract the ſecond; then to the Remainder add the Third, and the Reſult of theſe Operations will expreſs the Probability of the third Order.*

THE Probability of the fourth Order will be found thus.

1°. *Divide the ſhorteſt Complement by the double of the longeſt, and reſerve the Quotient.*

2°. *Divide the Square of the ſhorteſt Complement by three Times the Product of the longeſt and middlemoſt, and reſerve the new Quotient.*

3°. *From the firſt Quotient reſerved, ſubtract the ſecond, and the Remainder will expreſs the Probability of the fourth Order.*

THE fifth Order will be found as follows.

Divide the Square of the ſhorteſt Complement by ſix times the Product of the longeſt and middlemoſt, multiplied together, and the Quotient will expreſs the Probability required.

THE Probability of the laſt Order is the ſame as that of the fifth.

PROBLEM XVII.

D, *whilſt in Health, makes a Will, whereby he bequeaths* 500 l. *to* E, *and* 300 l. *to* F. *with this Condition, that if either of them dies before him, the whole is to go to the Survivor of the two; what are the Values of the Expectations of* E *and* F, *eſtimated from the time that the Will was writ.*

SOLUTION.

SUPPOSE *D* to be 70 Years of Age, *E* 36, and *F* 45; ſuppoſe alſo that *d* repreſents the Intereſt of 1 *l.* when Intereſt is at 5 *per Cent.*

AN Annuity upon the Life of *D* is worth 5.77, as appears from our Tables, which Value we may call *M*.

WHEREFORE if it was ſure that *D* would die before either of them, the Expectation of *E* upon that Account, would be worth in preſent Value $\overline{1-dM}\times 500$, and the Expectation of *F*, $\overline{1-dM}\times 300$; which being reduced to Numbers, are reſpectively 355 *l.* 15 *s.* and 213 *l.* 9 *s.*

BUT as this depends on the Probability of *D*'s dying firſt, we are to look for that Probability, which is compoſed of two Parts, that is, when the Order of Survivorſhip is either *E*, *F*, *D*, or *F*, *E*, *D*; now the Order *E*, *F*, *D*, is the ſame as *A*, *B*, *C*, in the preceding Problem, whereof the Probability is $1-\frac{p}{2n}-\frac{q}{2p}+\frac{qq}{9np}$, and the Order *F*, *E*, *D*, is the ſame as *B*, *A*, *C*, whereof the Probability is $\frac{p}{2p}-\frac{q}{2n}+\frac{qq}{6np}$, and the Sum of thoſe two Probabilities, *viz.* $1-\frac{q}{2p}-\frac{q}{2n}+\frac{qq}{3np}$, will expreſs the Probability of *D*'s dying before them both.

NOW the Ages being given, their Complements of Life will alſo be given, ſo that n will be found $=50$, $p=41$, $q=16$; for which reaſon the Probability juſt now ſet down being expreſſed in Numbers, will be 0.6865, and this being multiplied by the Expectations before found, *viz.* 315 *l.* 15 *s.* and 213 *l.* 9 *s.* will produce 244 *l.* 3*s.* 5*d.* and 146*l.* 10*s.* 8*d.* and theſe Sums expreſs the preſent Expectations of *E* and *F*, ariſing from the Proſpect of *D*'s dying before either of them.

BUT, both *E* and *F* have a farther Expectation; which, in reſpect to *E*, is, that he ſhall ſurvive *D*, and that *D* ſhall ſurvive *F*, in which Caſe he obtains 800 *l.* but this not being to be obtained before the Deceaſe of *D*, is reduced in preſent Value to 569*l.* 4*s.* Now the Probability of obtaining this anſwers to the Order, *A*, *C*, *B*, in the preceding Problem, which is expreſſed by $\frac{q}{2p} - \frac{qq}{3np} = 0.1535$; and therefore multiplying the Sum 569 *l.* 4*s.* by 0.1535, the Product will be 87*l.* 7*s.* 5*d.* and this will be the ſecond Part of *E*'s Expectation, which being joined with the firſt Part found before, *viz.* 244*l.* 3*s.* 5*d.* the Sum will be 331 *l.* 10 *s.* 10 *d.* which is the total Expectation of *E*, or the preſent Sum he might juſtly expect, if he would ſell his Right to another.

In the ſame manner the total Expectation of *F* will be found to be 213*l.* 18*s.* 6d.

Of the Expectations of Life.

I call that the Expectation of Life, the Time which a Perſon of a given

Age may juſtly expect to continue in being.

I have found by a Calculation deduced from the Method of Fluxions, that upon Suppoſition of an equable Decrement of Life, the Expectation of Life would be expreſſed by $\frac{1}{2}n$, ſuppoſing n to denote its Complement.

HOWEVER, if that Interval be once attained, there ariſes a new Expectation of $\frac{1}{4}n$, and afterwards of $\frac{1}{8}n$, and ſo on. This being laid down, I ſhall proceed farther.

PROBLEM XVIII.

To find the Expectation of two joint Lives, that is, the Time which two Lives may expect to continue together in being.

SOLUTION.

LET the Complements of the Lives be n and p, whereof n be the longeſt and p the ſhorteſt, then the Expectation of the two joint Lives, will be $\frac{1}{2}p - \frac{pp}{6n}$, in Words thus.

From $\frac{1}{2}$ the ſhorteſt Complement, ſubtract the 6th Part of its Square, divided by the greateſt, the Remainder will expreſs the Number of Years ſought.

THUS, ſuppoſing a Life of 40, and another of 50, the ſhorteſt Complement will be 36, the greateſt 46, $\frac{1}{2}$ of the ſhorteſt will be 18, the Square of 36 is 1296, whereof the ſixth Part is 216, which being divided by 46, the Quotient will be $\frac{216}{46} = 4.69$ nearly; and this being ſubtracted from 18, the Remainder 13.31 will expreſs the Number of Years due to the two joint Lives.

COROLLARY.

IF the two Lives be equal, the Expectation of the two joint Lives will be $\frac{1}{3}$ part of their common Complement.

PROBLEM XIX.

Any Number of Lives being given, whether equal or unequal, to find how many Years they may be expected to continue together.

SOLUTION.

1°. TAKE $\frac{1}{2}$ of the ſhorteſt Complement.

2°. TAKE $\frac{1}{6}$ part of the Square of the ſhorteſt, which divide ſucceſſively by all the other Complements, then add all the Quotients together.

3°. TAKE $\frac{1}{12}$ part of the Cube of the ſhorteſt Complement, which divide ſucceſſively by the Product of all the other Complements, taken two and two.

4°. THEN take $\frac{1}{20}$ part of the Biquadrate of the ſhorteſt Complement, which divide ſucceſſively by the Products of all the other Complements, taken three and three, and ſo on.

5°. THEN from the Reſult of the firſt Operation, ſubtract the Reſult of the ſecond, to the Remainder add the Reſult of the third, from the Sum ſubtract the Reſult of the fourth, and ſo on.

6°. THE laſt Quantity remaining after theſe alternate Subtractions and Additions, will be the thing required.

N. B. The Diviſors 2, 6, 12, 20, *&c.* are the Products of 1 by 2, of 2 by 3, of 3 by 4, of 4 by 5, *&c.*

COROLLARY.

IF all the Lives be equal, add Unity to the Number of Lives, and divide their common Complement by that Number thus increaſed by Unity, and the Quotient will always expreſs the Time due to their joint Continuance.

PROBLEM

PROBLEM XX.

Two Lives being given, to find the Number of Years due to the longeſt.

SOLUTION.

FROM the Sum of the Years due to each Life, ſubtract the Number of Years due to their joint Continuance, the Remainder will be the Number of Years due to the Longeſt, or Survivor of them both.

THUS, ſuppoſing a Life of 40, and another of 50, the Number of Years due to the Life of 40, is 23; the Number of years due to the Life of 50, is 18; from the Sum of 23 and 18, *viz.* 41, ſubtract 13.31 due to their joint Continuance, the Remainder 27.69 will be the Time due to the longeſt.

COROLLARY.

IF the Lives be equal, then $\frac{2}{3}$ of their common Complement will be the Number of Years due to the Survivor.

THUS, ſuppoſing two Lives of 50, then their Complement will be 36; whereof two thirds will be 24; which is the Time due to the Survivor of the two.

PROBLEM XXI.

Any Number of Lives being given, to find the Number of Years due to the Longeſt.

SOLUTION.

LET the Years due to each Life be reſpectively denoted by M, P, Q, S, *&c.* then let the joint Lives, taken two and two, be denoted by $\overline{MP}$, $\overline{MQ}$, $\overline{MS}$, $\overline{PQ}$, &c. let alſo the joint Lives, taken three and three, be denoted by $\overline{MPQ}$, $\overline{MPS}$, $\overline{MQS}$, $\overline{PQS}$, &c. Moreover, let the joint Lives, taken four and four, be denoted by $\overline{MPQS}$, &c. then if there be three Lives, the Time due to the longeſt will be

$$\begin{array}{l} M-\overline{MP}+\overline{MPQ} \\ +P-\overline{MQ} \\ +Q-\overline{PQ} \end{array}$$

BUT if all the Lives be equal, let n be their common Complement, then the Time due to the longeſt, will be $\frac{3}{4}n$.

IF there be four Lives, the Time due to the longeſt will be

$$\begin{array}{l} M-\overline{MP}+\overline{MPQ}-\overline{MPQS} \\ +P-\overline{MQ}+\overline{MPS} \\ +Q-\overline{MS}+\overline{MQS} \\ +S-\overline{PQ}+\overline{PQS} \\ \quad\;\; -\overline{PS} \\ \quad\;\; -\overline{QS} \end{array}$$

BUT if all the Lives be equal, the Time due to the longeſt will be expreſſed by $\frac{4}{5}$ of their common Complement.

Univerſally, if the Common Complement of equal Lives be n, and the Number of Lives p, the Number of

Years due to the Longeſt of them will be $\frac{p}{p+1}\times n$.

PROBLEM XXII.

Any Number of equal Lives being given, to find the Time in which one, or two, or three, &c. *of them will fail.*

SOLUTION.

LET n be their common Complement, p the Number of all the Lives, q the Number of thoſe which are to fail, then $\frac{q}{p+1}\times n$ will expreſs the Time required. In Words thus;

Multiply the common Complement of the Lives by the Number of the Lives that are to drop, and divide the Product by the Number of all the Lives increaſed by Unity.

THUS, ſuppoſing 100 Lives, each of 40 Years of Age, it will be found that 5 of them will drop in about two Years and a Quarter.

BUT if we put t for the Time given, we ſhall have the four following Equations;

$$1^{o}.\ t=\frac{qn}{p+1}$$

$$2^{o}.\ q=\frac{\overline{p+1}\times t}{n}$$

$$3^{o}.\ p=\frac{nq-t}{t}$$

$$4^{o}.\ n=\frac{\overline{p+1}\times t}{q}$$

In which any three of the four Quantities n, p, q, t, being given, the fourth will be known.

THIS Speculation might be carried to any Number of unequal Lives: but my Deſign not being to perplex the Reader with too great Difficulties, I ſhall forbear at preſent to proſecute the thing any farther.

PROBLEM XXIII.

A, *who is* 30 *Years of Age, buys an Annuity of* 1 l. *for a limited Time of his Life, ſuppoſe* 10 *Years, on Condition that if he dies before the Expiration of that Time, the Purchaſe-Money is wholly to be loſt to his Heirs; to find the preſent*

ſent Value of the Purchaſe, ſuppoſing Intereſt at 5 per Cent.

SOLUTION.

LET n be the Complement of *A*'s Life, m the limited Number of Years, p the Difference of n and m; Q the Value of an Annuity of 1 *l.* certain for m Years, and V the Value of the Perpetuity: then the preſent Value of the Purchaſe will be $\frac{mV + \overline{p - \overline{V+1}} \times Q}{n}$. In Words thus;

1°. *Multiply* V, *the Value of a Perpetuity, at the given Rate of Intereſt, by* m *the limited Number of Years, and reſerve the Product.*

2°. *To the ſame* V *add Unity, and take the Difference between their Sum and* p, *which is the Exceſs of the Complement of* A'*s Age above the limited Number of Years: multiply this Difference by* Q, *an Annuity certain for* m *Years, to get the ſecond Product.*

3°. *Let the* Sum *of theſe Products, if* p *is* greater *than* $V+1$; *and their* Difference, *if it is* leſſer, *be divided* *by* n, *the Complement of* A'*s Age; and the Quotient ſhall be the Value of the Purchaſe.*

As, in the Queſtion propoſed, where $n=56$, $m=10$, $p=46$, $Q=7.7212$, and $V=20$; the firſt Product (mV) is that of 20 by 10, or 200. And $p-\overline{V+1}$ being $46-21=25$, the ſecond Product is 25×7.7212, that is 193.0302. The two Products *added* (p being greater than $V+1$) make 393.0302; which divided by 56 quotes, for the Anſwer, 7.0184 Years Purchaſe.

NOTE, 1. When it happens that p is equal to $V+1$; as, Intereſt being at 5 *per Cent*, if the Difference of n and m is 21; the ſecond Product $\overline{p-\overline{V+1}}\times Q$ vaniſhing, the Anſwer is ſimply $\frac{mV}{n}$.

2. IF $m=n$, or $p=0$, ſeeing V is equal to $\frac{1}{r-1}$, the Expreſſion will be changed into $\frac{1}{r-1}+\frac{rQ}{n\times\overline{r-1}}$; which coincides with the Solution of *Prob.* I: Q repreſenting now the ſame Thing as P did in that Problem.

3. By this Propoſition, ſome uſeful Queſtions concerning *Inſurances* may be reſolved.

Suppose *A*, at 30 Years of Age, aſſigns over to *B* an Annuity of 1000 *l.* a Year, limited to 10 Years, and depending likewiſe upon *A*'s Life: then, by the foregoing Solution, *A* ought to receive for it only 7018 *l.* 8 *s.* Intereſt being at 5 *per Cent.* But if *B* wants that the Annuity ſhould ſtand clear of all Riſques, he muſt pay for it the Value *certain*, which is 7721*l.* 4 *s.* and *A* ought to have his Life inſured for 702 *l.* 16 *s.* the juſt Price of ſuch an Inſurance being *the Difference of the Values of the Annuity* certain, *and of the ſame Annuity* ſubject to the Contingency of the Annuitant's Life failing.

The ſame 702 *l.* 16 *s.* is likewiſe the Value of the Reverſion of this Annuity to a Perſon and his Heirs, who ſhould ſucceed to the Remainder of the 10 Years, upon *A*'s Deceaſe. *See* Prob. XXV.

IT is evident by the foregoing Proceſs, that altho' the Queſtion there propoſed is particular, yet the Solution is general; which Method, often practiſed in my *Doctrine of Chances*, is of ſingular Uſe to fix the Reader's Imagination.

PROBLEM XXIV.

A *pays an Annuity of* 100l. *during the Lives of* B *and* C, *each* 34 *Years of Age; to find what* A *ought to give in preſent Money to buy off the Life of* B, *ſuppoſing Intereſt at* 4 per Cent.

SOLUTION.

IT will be found by our Tables that an Annuity upon a Life of 34 is worth 14.12 Years Purchaſe; and, by the Rules before delivered, that an Annuity upon the longeſt of the two Lives of *B* and *C* is worth 18.40: hence it is very plain, that, to buy off the Life of *B*, *A* muſt pay the Difference between 18.40 and 14.12, which being 4.28, it follows that *A* ought to pay 428 *l.*

IN the ſame manner, if *A* were to pay an Annuity during the three Lives of *B*, *C*, *D*, whether of the ſame or different Ages, it would be eaſy to determine what *A* ought to pay to buy off one of the Lives of *B*, *C*, *D*, or any two of them, or to redeem the whole.

FOR, 1°. if the Life of *D* is to be bought off, then from the Value of the three Lives ſubtract the Value of the two Lives of *B* and *C*, and the Remainder is what is to be given to buy off the Life of *D*.

2°. IF the two Lives of *C* and *D* were to be bought off, then from the Value of the three Lives, ſubtract the Life of *B*, and the Remainder is what is to be given to buy off thoſe two Lives.

Laſtly, It is plain that to redeem the whole, the Value of the three Lives ought to be paid.

PROBLEM XXV.

A, *whoſe Life is worth* 14 *Years Purchaſe, ſuppoſing Intereſt at* 4 per Cent. *is to enjoy an Annuity of* 100l. *during the Term of* 31 *Years*; B *and his Heirs have the Reverſion of it after the Deceaſe*

ceaſe of A *for the Term remaining; to find the Value of* B's *Expectation.*

SOLUTION.

SINCE the Life of *A* is ſuppoſed to be worth 14 Years Purchaſe when Intereſt is at 4 *per Cent.* it follows from the Tables that *A* muſt be about 35 Years of Age, therefore find, by the twenty-third Propoſition, the Value of an Annuity on a Life of 35, to continue the limited Time of 31 Years; let that Value be ſubtracted from the Value of an Annuity certain, to continue 31 Years; and the Remainder will be the Value of the Reverſion.

PROBLEM XXVI.

A *is to have an Annuity of* 100 l. *for him and his Heirs after the failing of any one of the Lives* M, P, Q, *the firſt of which is worth* 13 *Years Purchaſe, the ſecond* 14, *and the third* 15; *to find the preſent Value of his Expectation, Intereſt of Money being ſuppoſed at* 4 per Cent.

SOLU-

SOLUTION.

BY the Example to Prob. 3. it appears, that an Annuity upon the above 3 joint Lives is worth 7.41 Years Purchaſe; let this be ſuppoſed $=R$, and let *ſ* repreſent the preſent Value of a Perpetuity of 100 *l.* which in this Caſe is 2500 *l.* then the preſent Expectation of *A* will be worth $\overline{1-dR}\times ſ$. In Words thus;

Multiply the Value of the three joint Lives by the Intereſt of 1 l. *then ſubtracting that Product from Unity, let the Remainder be multiplied by the Value of the Perpetuity, and the Product will be the Expectation required.*

IN this Caſe 7.41, multiplied by 0.04, produces 0.2964, and this Product ſubtracted from Unity, leaves 0.7036; now this Remainder being multiplied by 2500, produces 1759 *l.* the Expectation of *A*.

BUT if the Problem had been, that *A* ſhould not have the Annuity before the Failing of any two of thoſe Lives; from the Sum of all the joint Lives combined two and two, ſubtract the double Value of the three joint Lives, and let the Remainder be called *T*, then the Expectation of *A* will be worth $\overline{1-dT}\times ſ$; now, by the Rules before delivered, we ſhall find that the Sum of all the joint Lives combined two and two, is 29.06, from which ſubtracting the double of the three joint Lives, *viz.* 14.82, the Remainder is 14.24. Hence ſuppoſing $T=14.24$, then $\overline{1-dT}\times ſ$ will be found to be 1076*l.* and this is the Value of *A*'s Expectation.

Laſtly, If *A* was not to have the Annuity before the Extinction of the three Lives, ſuppoſe the Value of the three Lives $=V$, then the Expectation of *A* would be worth $\overline{1-dV}\times ſ$, which in this Caſe is 465*l.*

PROBLEM XXVII.

To determine the Fines to be paid for renewing any Number of Years in a College-Leaſe of twenty; and alſo what

Rate of Intereſt is made by a Purchaſer, who may happen to give an advanced Price for the ſame, upon Suppoſition that the Contractor is allowed 8 per Cent. *of his Money.*

ALTHO' the Problem here propoſed does not ſeem to relate to the Subject of this Book, yet as ſome uſeful Concluſions may be derived from the Solution of it, I have thought fit to inſert it in this Place.

Table of Fines.

1	0.2146	8	2.2821	15	5.8254
2	0.4463	9	2.6792	16	6.5060
3	0.6965	10	3.1081	17	7.2411
4	0.9666	11	3.5713	18	8.0349
5	1.2587	12	4.0715	19	8.9922
6	1.4133	13	4.6118	20	9.8181
7	1.9144	14	5.1953		

IF a Purchaſer gives the Original Contractor 11 Years Purchaſe for his Leaſe of 20, he makes above 6½ *per Cent.* of his Money.

IF he gives 12 Years Purchaſe for the ſame, he makes above 5 *l.* 8 *s.* *per Cent.* of his Money.

IF he gives 13 Years Purchaſe, he makes $4\frac{1}{2}$ *per Cent.* of his Money.

PROBLEM XXVIII.

To determine the Fines to be paid for renewing any Number of Years in a College-Leaſe of One and Twenty; as alſo what Rate of Intereſt is made by a Purchaſer who may happen to give an advanced Price for the ſame, upon Suppoſition that the Contractor is allowed 8 per Cent. *of his Money.*

Table of Fines.

1	0.1987	8	2.1131	15	5.3940
2	0.4133	9	2.2808	16	6.0241
3	0.6450	10	2.8779	17	6.7047
4	0.8952	11	3.3068	18	7.4398
5	1.1653	12	3.7700	19	8.2336
6	1.4574	13	4.2702	20	9.0909
7	1.6120	14	4.8105	21	10.0168

HE that gives 11 Years Purchaſe, inſtead of 10.0168 for renewing his Leaſe for 21 Years, makes 6 *l.* 16 *s.* *per Cent.* of his Money.

HE who gives 12 Years Purchaſe for the ſame, makes very near 5 *l.* 16 *s.* *per Cent.* of his Money.

HE who gives 13 Years Purchaſe for the ſame, makes a little more than 4*l.* 16*s.* *per Cent.* of his Money.

The Values of Annuities for Lives having been calculated, in this Book, upon a Suppoſition that the Payments are made Yearly, and there being ſome Occaſions wherein it is ſtipulated that the Payments ſhould be made Half-Yearly, I have thought fit to add the two following Problems; *whereby,* 1°. *It is ſhewn what the Half-Yearly Payments ought to be, if the Price of the Purchaſe is preſerved.* 2°. *How the Price of the Purchaſe ought to be increaſed, if the Half-Yearly Payments are required to be the Half of the Yearly Payments.*

PROBLEM XXIX.

An Annuity being given, to find what Half-Yearly Payments will be equivalent to it, when Intereſt of Money is 4, 5, *or* 6 per Cent.

SOLUTION.

TAKE Half of the Annuity, and from that Half ſubtract its 100th, or 80th, or 68th Part, according as the Intereſt is 4, 5, or 6 *per Cent.* and the Remainder will be the Value of the Half-Yearly Payments required; thus, if the Annuity was 100*l.* the Half-Yearly Payments would reſpectively be 49*l.* 10*s.* 49*l.* 7*s.* 6*d.* 49*l.* 5*s.* 3*d.* nearly.

PROBLEM XXX.

The preſent Value of an Annuity being given, to find how much this preſent Value ought to be increaſed, when it is required that the Payments ſhall be Half-Yearly, and alſo one Half of the Yearly Payments, when Intereſt is at 4, 5, *or* 6 per Cent.

SOLUTION.

To the preſent Value of the Annuity add reſpectively its 99th, 79th, or 67th, and the Sums will be the Values increaſed.

As there are ſome Perſons who may be deſirous to ſee a general Solution of the two laſt Problems, I have thought fit to add what follows.

IN the firſt of the two laſt Problems, let *A* be the Yearly Payments agreed on, and *B* the Half-Yearly Payments required, *r* the Yearly Rate of Intereſt, then $B = \frac{r^{\frac{1}{2}} - 1}{r - 1} \times A$. In the ſecond, let *M* be the preſent Value of the Yearly Payments, *P* the preſent Value of thoſe that are to be Half-Yearly, then $P = \frac{\frac{1}{2} \times \overline{r - 1}}{r^{\frac{1}{2}} - 1} \times M$.

TABLE I.

The preſent Value of an Annuity of one Pound, for any Number of Years not exceeding 100, *Intereſt at* 3 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9709	26	17.8768	51	25.9512	76	29.8076
2	1.9135	27	18.3270	52	26.1662	77	29.9103
3	2.8286	28	18.7641	53	26.3750	78	30.0100
4	3.7170	29	19.1884	54	26.5777	79	30.1068
5	4.5797	30	19.6004	55	26.7744	80	30.2008
6	5.4172	31	20.0004	56	26.9655	81	30.2920
7	6.2303	32	20.3887	57	27.1509	82	30.3806
8	7.0197	33	20.7658	58	27.3310	83	30.4666
9	7.7861	34	21.1318	59	27.5058	84	30.5501
10	8.5302	35	21.4872	60	27.6756	85	30.6311
11	9.2526	36	21.8323	61	27.8404	86	30.7099
12	9.9540	37	22.1672	62	28.0003	87	30.7863
13	10.6350	38	22.4925	63	28.1557	88	30.8605
14	11.2961	39	22.8082	64	28.3065	89	30.9325
15	11.9379	40	23.1148	65	28.4529	90	31.0024
16	12.5611	41	23.4124	66	28.5950	91	31.0703
17	13.1611	42	23.7014	67	28.7330	92	31.1362
18	13.7535	43	23.9819	68	28.8670	93	31.2001
19	14.3238	44	24.2543	69	28.9971	94	31.2622
20	14.8775	45	24.5187	70	29.1234	95	31.3224
21	15.4150	46	24.7754	71	29.2460	96	31.3809
22	15.9369	47	25.0247	72	29.3651	97	31.4377
23	16.4436	48	25.2667	73	29.4807	98	31.4928
24	16.9355	49	25.5017	74	29.5929	99	31.5463
25	17.4131	50	25.7298	75	29.7018	100	31.5984

The Value of the Perpetuity is 33 $\frac{1}{3}$ *Years Purchaſe.*

TABLE II.

The preſent Value of an Annuity of one Pound, to continue ſo long as a Life of a given Age is in being, Intereſt being eſtimated at 3 per Cent.

Age.	Value	Age.	Value	Age.	Value	Age.	Value
1	15.05	26	17.50	51	12.26	76	4.05
2	16.62	27	17.33	52	12.00	77	3.63
3	17.83	28	17.16	53	11.73	78	3.21
4	18.46	29	16.98	54	11.46	79	2.78
5	18.90	30	16.80	55	11.18	80	2.34
6	19.33	31	16.62	56	10.90	81	1.89
7	19.60	32	16.44	57	10.61	82	1.43
8	19.74	33	16.25	58	10.32	83	0.96
9	19.87	34	16.06	59	10.03	84	0.49
10	19.87	35	15.86	60	9.73	85	0.00
11	19.74	36	15.67	61	9.42	86	0.00
12	19.60	37	15.46	62	9.11		
13	19.47	38	15.26	63	8.79		
14	19.33	39	15.05	64	8.46		
15	19.19	40	14.84	65	8.13		
16	19.05	41	14.63	66	7.79		
17	18.90	42	14.41	67	7.45		
18	18.76	43	14.19	68	7.10		
19	18.61	44	13.96	69	6.75		
20	18.46	45	13.73	70	6.38		
21	18.30	46	13.49	71	6.01		
22	18.15	47	13.25	72	5.63		
23	17.99	48	13.01	73	5.25		
24	17.83	49	12.76	74	4.85		
25	17.66	50	12.51	75	4.45		

TABLE III.

The preſent Value of an Annuity of one Pound, for any Number of Years not exceeding 100, *Intereſt at* 3½ per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9662	26	16.8904	51	23.6286	76	26.4799
2	1.8997	27	17.2854	52	23.7958	77	26.5506
3	2.8016	28	17.6670	53	23.9573	78	26.6190
4	3.6731	29	18.0358	54	24.1133	79	26.6850
5	4.5151	30	18.3920	55	24.2641	80	26.7488
6	5.3286	31	18.7363	56	24.4097	8	26.8104
7	6.1145	32	19.0689	57	24.5504	82	26.8700
8	6.8740	33	19.3902	58	24.6864	83	26.9275
9	7.6077	34	19.7007	59	24.8178	84	26.9831
10	8.3166	35	20.0007	60	24.9447	85	27.0368
11	9.0015	36	20.2905	61	25.0674	86	27.0887
12	9.6633	37	20.5705	62	25.1859	87	27.1388
13	10.3027	38	20.8411	63	25.3004	88	27.1873
14	10.9205	39	2 .1025	64	25.4110	89	27.2341
15	11.5174	40	21.3551	65	25.5178	90	27.2793
16	12.0941	41	21.5991	66	25.6211	91	27.3230
17	12.6513	42	21.8349	67	25.7209	92	27.3652
18	13.1897	43	22.0627	68	25.8173	93	27.4060
19	13.7098	44	22.2828	69	25.9104	94	27.4454
20	14.2124	45	22.4955	70	26.0004	95	27.4835
21	14.6980	46	22.7009	71	26.0873	96	27.5203
22	15.1671	47	22.8994	72	26.1713	97	27.5558
23	15.6204	48	23.0912	73	26.2525	98	27.5902
24	16.0584	49	23.2766	74	26.3309	99	27.6234
25	16.4815	50	23.4556	75	26.4067	100	27.6554

The Value of the Perpetuity is 28 $\frac{4}{7}$ *Years Purchaſe.*

TABLE IV.

The preſent Value of an Annuity of one Pound, ſo long as a Life of a given Age is in being, Intereſt being eſtimated at 3½ per Cent.

Age.	Value	Age.	Value	Age.	Value	Age.	Value
1	14.16	26	16.28	51	11.69	76	3.98
2	15.53	27	16.13	52	11.45	77	3.57
3	16.56	28	15.98	53	11.20	78	3.16
4	17.09	29	15.83	54	10.95	79	2.74
5	17.46	30	15.68	55	10.69	80	2.31
6	17.82	31	15.53	56	10.44	81	1.87
7	18.05	32	15.37	57	10.18	82	1.42
8	18.16	33	15.21	58	9.91	83	0.95
9	18.27	34	15.05	59	9.64	84	0.48
10	18.27	35	14.89	60	9.36	85	0.00
11	18.16	36	14.71	61	9.08	86	0.00
12	18.05	37	14.52	62	8.79		
13	17.94	38	14.34	63	8.49		
14	17.82	39	14.16	64	8.19		
15	17.71	40	13.98	65	7.88		
16	17.59	41	13.79	66	7.56		
17	17.46	42	13.59	67	7.24		
18	17.33	43	13.40	68	6.91		
19	17.21	44	13.20	69	6.57		
20	17.09	45	12.99	70	6.22		
21	16.96	46	12.78	71	5.87		
22	16.83	47	12.57	72	5.51		
23	16.69	48	12.36	73	5.14		
24	16.56	49	12.14	74	4.77		
25	16.42	50	11.92	75	4.38		

TABLE V.

The preſent Value of an Annuity of one Pound, for any Number of Years not exceeding 100, *Intereſt at* 4 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9615	26	15.9827	51	21.6714	76	23.7311
2	1.8860	27	16.3295	52	21.7475	77	23.7799
3	2.7750	28	16.6630	53	21.8726	78	23.8268
4	3.6298	29	16.9837	54	21.9929	79	23.8720
5	4.4518	30	17.2920	55	22.1086	80	23.9153
6	5.2421	31	17.5884	56	22.2198	81	23.9571
7	6.0020	32	17.8735	57	22.3267	82	23.9972
8	6.7327	33	18.1476	58	22.4295	83	24.0357
9	7.4353	34	18.4111	59	22.5284	84	24.0728
10	8.1108	35	18.6646	60	22.6234	85	24.1085
11	8.7604	36	18.9082	61	22.7148	86	24.1428
12	9.3850	37	19.1425	62	22.8027	87	24.1757
13	9.9856	38	19.3678	63	22.8872	88	24.2074
14	10.5631	39	19.5844	64	22.9685	89	24.2379
15	11.1183	40	19.7927	65	23.0466	90	24.2672
16	11.6522	41	19.9930	66	23.1218	91	24.2954
17	12.1656	42	20.1856	67	23.1940	92	24.3225
18	12.6592	43	20.3707	68	23.2635	93	24.3486
19	13.1339	44	20.5488	69	23.3302	94	24.3736
20	13.5903	45	20.7200	70	23.3945	95	24.3977
21	14.0291	46	20.8846	71	23.4562	96	24.4209
22	14.4511	47	21.0429	72	23.5156	97	24.4431
23	14.8568	48	21.1951	73	23.5727	98	24.4646
24	15.2469	49	21.3414	74	23.6276	99	24.4851
25	15.6220	50	21.4821	75	23.6804	100	24.5049

TABLE VI.

The preſent Value of an Annuity of one Pound, to continue ſo long as a Life of a given Age is in being, Intereſt being eſtimated at 4 per Cent.

Age.	Value	Age.	Value	Age.	Value	Age.	Value
1	13.36	26	15.19	51	11.13	76	3.91
2	14.54	27	15.04	52	10.92	77	3.52
3	15.43	28	14.94	53	10.70	78	3.11
4	15.89	29	14.81	54	10.47	79	2.70
5	16.21	30	14.68	55	10.24	80	2.28
6	16.50	31	14.54	56	10.01	81	1.85
7	16.64	32	14.41	57	9.77	82	1.40
8	16.79	33	14.27	58	9.52	83	0.95
9	16.88	34	14.12	59	9.27	84	0.48
10	16.88	35	13.98	60	9.01	85	0.00
11	16.79	36	13.82	61	8.75	86	0.00
12	16.64	37	13.67	62	8.48		
13	16.60	38	13.52	63	8.20		
14	16.50	39	13.36	64	7.92		
15	16.41	40	13.20	65	7.63		
16	16.31	41	13.02	66	7.33		
17	16.21	42	12.85	67	7.02		
18	16.10	43	12.68	68	6.75		
19	15.99	44	12.50	69	6.39		
20	15.89	45	12.32	70	6.06		
21	15.78	46	12.13	71	5.72		
22	15.67	47	11.94	72	5.38		
23	15.55	48	11.74	73	5.02		
24	15.43	49	11.54	74	4.66		
25	15.31	50	11.34	75	4.29		

TABLE VII.

The preſent Value of an Annuity of one Pound, for any Number of Years not exceeding 100, *Intereſt at* 5 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9523	26	14.3751	51	18.3389	76	19.5094
2	1.8594	27	14.6430	52	18.4180	77	19.5328
3	2.7232	28	14.8981	53	18.4934	78	19.5550
4	3.5459	29	15.1410	54	18.5651	79	19.5762
5	4.3294	30	15.3724	55	18.6334	80	19.5964
6	5.0756	31	15.5928	56	18.6985	81	19.6156
7	5.7863	32	15.8026	57	18.7605	82	19.6339
8	6.4632	33	16.0025	58	18.8195	83	19.6514
9	7.1078	34	16.1929	59	18.8757	84	19.6680
10	7.7212	35	16.3741	60	18.9292	85	19.6838
11	8.3064	36	16.5468	61	18.9802	86	19.6988
12	8.8632	37	16.7112	62	19.0288	87	19.7132
13	9.3935	38	16.8678	63	19.0750	88	19.7268
14	9.8986	39	17.0170	64	19.1191	89	19.7398
15	10.3796	40	17.1590	65	19.1610	90	19.7522
16	10.8377	41	17.2943	66	19.2010	91	19.7640
17	11.2740	42	17.4232	67	19.2390	92	19.7752
18	11.6895	43	17.5459	68	19.2753	93	19.7859
19	12.0853	44	17.6627	69	19.3098	94	19.7961
20	12.4622	45	17.7740	70	19.3426	95	19.8058
21	12.8211	46	17.8800	71	19.3739	96	19.8151
22	13.1630	47	17.9810	72	19.4037	97	19.8239
23	13.4885	48	18.0771	73	19.4321	98	19.8323
24	13.7986	49	18.1687	74	19.4592	99	19.8403
25	14.0939	50	18.2559	75	19.4849	100	19.8479

TABLE

TABLE VIII.

The preſent Value of an Annuity of one Pound, to continue ſo long as a Life of a given Age is in being, Intereſt at 5 per Cent.

Age.	Value	Age.	Value	Age.	Value	Age.	Value
1	11.96	26	13.37	51	10.17	76	3.78
2	12.88	27	13.28	52	9.99	77	3.41
3	13.55	28	13.18	53	9.82	78	3.03
4	13.89	29	13.09	54	9.63	79	2.64
5	14.12	30	12.99	55	9.44	80	2.23
6	14.34	31	12.88	56	9.24	81	1.81
7	14.47	32	12.78	57	9.04	82	1.38
8	14.53	33	12.67	58	8.83	83	0.94
9	14.60	34	12.56	59	8.61	84	0.47
10	14.60	35	12.45	60	8.39	85	0.00
11	14.53	36	12.33	61	8.16	86	0.00
12	14.47	37	12.21	62	7.93		
13	14.41	38	12.09	63	7.68		
14	14.34	39	11.96	64	7.43		
15	14.27	40	11.83	65	7.18		
16	14.20	41	11.70	66	6.91		
17	14.12	42	11.57	67	6.64		
18	14.05	43	11.43	68	6.36		
19	13.97	44	11.29	69	6.97		
20	13.89	45	11.14	70	5.77		
21	13.81	46	10.99	71	5.47		
22	13.72	47	10.84	72	5.15		
23	13.64	48	10.68	73	4.82		
24	13.55	49	10.51	74	4.49		
25	13.46	50	10.35	75	4.14		

TABLE IX.

The preſent Value of an Annuity of one Pound, for any Number of Years not exceeding 100, *Intereſt at* 6 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9433	26	13.0031	51	15.8130	76	16.4677
2	1.8333	27	13.2105	52	15.8613	77	16.4790
3	2.6730	28	13.4061	53	15.9069	78	16.4896
4	3.4651	29	13.5907	54	15.9499	79	16.4996
5	4.2123	30	13.7648	55	15.9905	80	16.5091
6	4.9173	31	13.9290	56	16.0288	81	16.5180
7	5.5823	32	14.0840	57	16.0649	82	16.5264
8	6.2097	33	14.2302	58	16.0989	83	16.5343
9	6.8016	34	14.3681	59	16.1311	84	16.5418
10	7.3600	35	14.4982	60	16.1614	85	16.5489
11	7.8868	36	14.6209	61	16.1900	86	16.5556
12	8.3838	37	14.7367	62	16.2170	87	16.5618
13	8.8526	38	14.8460	63	16.2424	88	16.5678
14	9.2949	39	14.9490	64	16.2664	89	16.5734
15	9.7122	40	15.0462	65	16.2891	90	16.5786
16	10.1058	41	15.1380	66	16.3104	91	16.5836
17	10.4772	42	15.2245	67	16.3306	92	16.5883
18	10.8276	43	15.3061	68	16.3496	93	16.5928
19	11.1581	44	15.3831	69	16.3676	94	16.5969
20	11.4699	45	15.4558	70	16.3845	95	16.6009
21	11.7640	46	15.5243	71	16.4005	96	16.6046
22	12.0415	47	15.5890	72	16.4155	97	16.6081
23	12.3033	48	15.6500	73	16.4297	98	16.6114
24	12.5503	49	15.7075	74	16.4431	99	16.6145
25	12.7833	50	15.7618	75	16.4558	100	16.6175

TABLE

TABLE X.

The preſent Value of an Annuity of one Pound, to continue ſo long as a Life of a given Age is in being, Intereſt being eſtimated at 6 per Cent.

Age.	Value	Age.	Value	Age.	Value	Age.	Value
1	10.80	26	11.90	51	9.34	76	3.66
2	11.53	27	11.83	52	9.20	77	3.31
3	12.04	28	11.76	53	9.04	78	2.95
4	12.30	29	11.68	54	8.90	79	2.57
5	12.47	30	11.61	55	8.72	80	2.18
6	12.63	31	11.53	56	8.56	81	1.78
7	12.74	32	11.45	57	8.38	82	1.36
8	12.79	33	11.36	58	8.20	83	0.92
9	12.84	34	11.60	59	8.02	84	0.77
10	12.84	35	11.18	60	7.83	85	0.00
11	12.79	36	11.09	61	7.63	86	0.00
12	12.74	37	11.00	62	7.42		
13	12.69	38	10.90	63	7.21		
14	12.63	39	10.80	64	7.00		
15	12.58	40	10.70	65	6.77		
16	12.53	41	10.60	66	6.53		
17	12.47	42	10.50	67	6.22		
18	12.41	43	10.37	68	6.03		
19	12.36	44	10.26	69	5.77		
20	12.30	45	10.14	70	5.50		
21	12.23	46	10.02	71	5.22		
22	12.17	47	9.90	72	4.93		
23	12.11	48	9.76	73	4.63		
24	12 04	49	9.63	74	4.32		
25	11.97	50	9.49	75	4.00		

Note; The 1*ſt*, 3*d*, 5*th*, 7*th*, and 9*th* Tables ſerve likewiſe as Tables of *Diſcount*; that is, for finding the preſent Value of a Sum payable after a given Time.

Thus, *to find the preſent Value of* 1000 l. *payable ſeven Years hence, at* 3½ per Cent. *Intereſt*; from the preſent Value of an Annuity of 1 *l.* certain for ſeven Years, which, in *Tab.* III. is 6.1145. I ſubtract the like Value for ſix Years, and the Remainder .7859 is the Value of the ſeventh Year's Rent, or of 1 *l.* payable after ſeven Years; which multiplied by 1000 gives the Anſwer 785 *l.* 18 *s.*

I may add, that they ſerve alſo *as Tables of Compound Intereſt*. For if it be aſked, what will be the Amount of the Sum *S* in 7 Years at $3\frac{1}{2}$ per Cent.? having found .7859 as above, 'tis plain the Amount will be $\frac{1}{.7859}\times S$.

Or if it be aſked, in what Time a Sum *S* will be doubled, tripled, &c. at 2, 3, &c. per Cent. compound Intereſt? I take, in the proper Table, two contiguous Numbers whoſe Difference is neareſt the Reciprocal of the *Rate* given, as $\frac{1}{2}$, $\frac{1}{3}$, &c. And the Year againſt the higher Number is the Anſwer.

Thus in Tab. I. for 3 per Cent. againſt the Years 22, 23 ſtand the Numbers 15.9369 and 16.4436, whoſe Difference .5067, being a little more than .5 or $\frac{1}{2}$, ſhews that in 23 Years the Sum will be a little more than doubled. And againſt the Years 36 and 37 are 21.8323 and 22.1672, the Difference whereof being .3349 (or nearly $\frac{1}{3}$) ſhews that in 14 Years more *S* is tripled.

Laſtly, If I would know at what *Rate* of Intereſt I ought to lay out a Sum *S*, ſo as it may increaſe $\frac{1}{3}$, or become $\frac{4}{3}$ S, in about 7 Years. Here the Fraction I am to look for is $\frac{3}{4}$, or the Decimal .75, which is not to be found in Tab. I. or III, till after the limited Time of 7 Years. But in Tab. V. the Numbers againſt 6 and 7 Years give the Difference .7599, and the Rate is 4 per Cent, nearly.

In like Manner may moſt uſeful Queſtions of this Kind be reſolved by theſe few compendious Tables.

APPEN-

APPENDIX:

Containing the Demonſtrations of ſome of the principal Propoſitions in the foregoing Treatiſe.

CHAPTER I.

I Obſerved formerly, that upon Suppoſition that the Decrements of Life were in Arithmetic Progreſſion, the Concluſions derived from thence would very little vary from thoſe, that could be deduced from the Table of Obſervations made at *Breſlaw*, concerning the Mortality of Mankind; which Table was about fifty Years ago inſerted by Dr. *Halley* in the *Philoſophical Tranſactions*, together with ſome Calculations concerning the Values of Lives according to a given Age.

UPON the foregoing Principle, I ſuppoſed that if *n* repreſented the Complement of Life, the Probabilities of living 1, 2, 3, 4, 5, &c. Years, would be expreſſed by the following Series, $\frac{n-1}{n}$, $\frac{n-2}{n}$, $\frac{n-3}{n}$, $\frac{n-4}{n}$, $\frac{n-5}{n}$, &c. and conſequently that the Value of a Life, whoſe Complement is n, would be expreſſed by the Series

$\frac{n-1}{nr} + \frac{n-2}{nrr} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5}$, &c. the Sum of which I have aſſerted in my 15th Page to be $\frac{1-\frac{r}{n}P}{r-1}$, where the Signification of the Quantities P and r is explained.

As the Reaſonings that led me to that general Expreſſion, require ſomething more than an ordinary Skill in the Doctrine of Series, I ſhall forbear to mention them in this Place; and content my ſelf with pointing out to the Reader a Method, whereby he may ſatisfy himſelf of the Truth of that Theorem, provided he underſtand ſo much of a Series, as to be able to ſum up a Geometric Progreſſion.

De-

DEMONSTRATION.

$$P = \frac{1}{r} + \frac{1}{rr} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} \cdots \frac{1}{r^n}.$$

Therefore,

$$rP = 1 + \frac{1}{r} + \frac{1}{rr} + \frac{1}{r^3} + \frac{1}{r^4} \cdots + \frac{1}{r^{n-1}}.$$

And

$$\frac{rP}{n} = \frac{1}{n} + \frac{1}{nr} + \frac{1}{nrr} + \frac{1}{nr^3} + \frac{1}{nr^4} \cdots + \frac{1}{nr^{n-1}}.$$

Therefore,

$$1 - \frac{rP}{n} = \frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nrr} - \frac{1}{nr^3} - \frac{1}{nr^4} - \cdots \frac{1}{nr^{n-1}}.$$

But this is to be divided by $r-1$, or multiplied by

$$\frac{1}{r-1} = \frac{1}{r} + \frac{1}{rr} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6}, \text{ \&c.}$$

Then multiplying actually thoſe two Series's together, the Product will be found to be

$$\begin{array}{l}
\frac{n-1}{nr} - \frac{1}{nrr} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.} \\
\qquad + \frac{n-1}{nrr} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.} \\
\qquad\qquad + \frac{n-1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.} \\
\qquad\qquad\qquad + \frac{n-1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.} \\
\qquad\qquad\qquad\qquad + \frac{n-1}{nr^5} - \frac{1}{nr^6} \text{ \&c.} \\
\qquad\qquad\qquad\qquad\qquad + \frac{n-1}{nr^6} \text{ \&c.}
\end{array}$$

And adding the Terms of the perpendicular Columns together, we ſhall have

$$\frac{n-1}{nr}+\frac{n-2}{nrr}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\frac{n-5}{nr^5}+\frac{n-6}{nr^6} \text{ \&c.}$$

which conſequently is equal to $\frac{1-\frac{r}{n}P}{r-1}$: which was to be demonſtrated.

IF it be required that upon the Failing of a Life, ſuch Part of the Annuity ſhould be paid, as may be proportional to the Time elapſed from the Beginning of the laſt Year, to the Time of the Life's failing, then the Value of the Life will be $\frac{1}{r-1}-\frac{1}{an}P$, wherein a repreſents the hyperbolic Logarithm of the Rate of Intereſt.

BUT becauſe there are no Tables printed of hyperbolic Logarithms, and that the Reduction of a common Logarithm to an hyperbolic is ſomewhat laborious, it will be ſufficient here to ſet down the hyperbolic Logarithms of 1.03, 1.035, 1.04, 1.05, 1.06, which are reſpectively, 0.02956, 0.0344, 0.03922, 0.04879, 0.05825, or $\frac{1}{35}$, $\frac{1}{31}$, $\frac{2}{51}$ $\frac{2}{41}$, $\frac{6}{103}$ nearly.

CHAPTER

CHAPTER II.

Explaining the Rules of combined Lives.

SUPPOSING a fictitious Life, whoſe Number of Chances to continue in being from Year to Year, are conſtantly equal to *a*, and the Number of Chances for failing are conſtantly equal to *b*, ſo that the Odds of its continuing during the Space of any one Year, be to its failing in the ſame Interval of Time conſtantly as *a* to *b*, the Value of an Annuity upon ſuch a Life would be eaſily found.

FOR, if we make $a+b=s$, the Probabilities of living 1, 2, 3, 4, 5, *&c.* Years would be repreſented by the Series $\frac{a}{s}$, $\frac{aa}{ss}$, $\frac{a^3}{s^3}$, $\frac{a^4}{s^4}$, $\frac{a^5}{s^5}$, *&c.* continued to Eternity; and conſequently the Value of an Annuity upon ſuch Life would be expreſſed by this new Series $\frac{a}{sr}+\frac{aa}{ssrr}+\frac{a^3}{s^3r^3}+\frac{a^4}{s^4r^4}$ *&c.* which being a geometric Progreſſion perpetually decreaſing, the Sum of it will be found to be $\frac{a}{sr-a}$: thus, if *a* ſtands for 21, and *b* for 1, and alſo *r* for 1.05, the Value of ſuch Life would be ten Years Purchaſe.

FROM theſe Premiſes the following Corollaries may be drawn:

COROLLARY I.

AN Annuity upon a fictitious Life being given, the Probability of its continuing one Year in being is alſo given; for let the Value be $= M$, then $\frac{a}{s} = \frac{Mr}{M+1}$.

COROLLARY II.

IF a Life, whoſe Value is deduced from our Tables, is found to be worth 10 Years Purchaſe, then ſuch Life is equivalent to a fictitious Life, whoſe Number of Chances for continuing one Year, is to the Number of Chances for its failing in that Year, as 21 to 1.

COROLLARY III.

WHEREFORE having taken the Value of a Life from our Tables, or calculated it according to the Rules preſcribed; we may transfer the Value of that Life to that of a fictitious Life, and find the Number of Chances it would have for continuing or failing Yearly.

COROLLARY IV.

AND the Combination of two or more *real Lives* will be very near the ſame as the Combination of ſo many correſponding *fictitious Lives*; and therefore an Annuity granted upon one or more real Lives, is nearly of the ſame Value as an Annuity upon a fictitious Life.

THESE things being premiſed, it will not be difficult to determine the Value of an Annuity upon two or three, or as many joint Lives as may be aſſigned.

FOR let x repreſent the Probability of one Life's continuing from Year to Year, and y the Probability of another Life's continuing the ſame Time; then according to the Principles of the Doctrine of Chances, the Terms

xy, $xxyy$, x^3y^3, x^4y^4, x^5y^5, &c.

will reſpectively repreſent the Probabilities of continuing together, 1, 2, 3, 4, 5, &c. Years; and the Value of an Annuity upon the two joint Lives, will be $\frac{xy}{r}+\frac{xxyy}{rr}+\frac{x^3y^3}{r^3}+\frac{x^4y^4}{r^4}+\frac{x^5y^5}{r^5}$ &c. which being a Geometrical Progreſſion perpetually decreaſing, the Sum of it will be found to be $\frac{xy}{r-xy}$: let now M be put for the Value of the firſt Life, and P for the Value of the ſecond, then by our firſt Corollary it appears that $x=\frac{Mr}{M+1}$, and $y=\frac{Pr}{P+1}$; and therefore having written theſe Values of x and y in the Expreſſion $\frac{xy}{r-x}$, which is the Value of the two joint Lives, it will be changed into $\frac{MPr}{\overline{M+1}\times\overline{P+1}-MPr}$: which is the ſame Theorem that I had given in my firſt Edition, which came out ſeventeen Years ago.

It is true that in the ſixth Page of this Book I have given a Theorem which ſeems very different from this; making the Value of the joint Lives to be $\frac{MP}{M+P-dMP}$, wherein d repreſents the Intereſt of 1 *l.* and yet I may aſſure the Reader, that this laſt Expreſſion is originally derived from the firſt; and that whether one or the other is uſed, the Concluſions will very little differ: but the firſt Theorem is better adapted to Annuities paid in Money, it being cuſtomary that the laſt Payment, whether it be Yearly or Half-Yearly, is loſt to the Purchaſer; whereas the ſecond Theorem is better fitted to Annuities paid by a Grant of Lands, whereby the Purchaſer makes Intereſt of his Money to the laſt Moment of his Life: for which Reaſon I have choſe to uſe the laſt Expreſſion in my Book.

By following the ſame Method of Inveſtigation, we ſhall find that if M, P, Q, denote three ſingle Lives, an

Annuity upon thoſe joint Lives will be $\frac{MPQrr}{\overline{M+1}\times\overline{P+1}\times\overline{Q+1}-MPQrr}$, in the Caſe of Annuities payable in Money; or $\frac{MPQ}{MP+MQ+PQ-2dMP}$, in the Caſe of Annuities paid by a Grant of Lands.

CHAPTER III.

Containing the Demonſtration of the Rules given in Problems 4th and 5th, for determining the longeſt Life.

LET x and y repreſent the reſpective Probabilities which two Lives have of continuing one Year in being, therefore $1-x$ is the Probability of the firſt Life's failing in one Year, and $1-y$ the Probability of the ſecond Life's failing in one Year: Therefore multiplying theſe two Probabilities together, the Product $1-x-y+xy$ will repreſent the Probability of the two Lives failing in one Year; and if this be ſubtracted from Unity, the Remainder $x+y-xy$ will expreſs the Probability of one at leaſt of the two Lives outliving one Year: which is ſufficient for eſtabliſhing the firſt Year's Rent.

AND, for the ſame Reaſon $xx+yy-xxyy$ will expreſs the Probability of one at leaſt of the two Lives outliving two Years: which is ſufficient to eſtabliſh the ſecond Year's Rent.

FROM the two Steps we have taken, it plainly appears that the longeſt of two Lives is expreſſible by the three following Series;

$$\frac{x}{r}+\frac{xx}{rr}+\frac{x^3}{r^3}+\frac{x^4}{r^4}+\frac{x^5}{r^5}$$

$$+\frac{y}{r}+\frac{yy}{rr}+\frac{y^3}{r^3}+\frac{y^4}{r^4}+\frac{y^5}{r^5}$$

$$-\frac{xy}{r}-\frac{xxyy}{rr}-\frac{x^3y^3}{r^3}-\frac{x^4y^4}{r^4}-\frac{x^5y^5}{r^3}$$

Whereof the firſt repreſents an Annuity upon the firſt Life, the ſecond an Annuity upon the ſecond Life, and the third an Annuity upon the two joint Lives; and therefore we may conclude that an Annuity upon the longeſt of two Lives, is the Difference between the Sum of the Values of the ſingle Lives, and the Value of the joint Lives: which has been expreſſed in my 14th Page by the Symbols $M+P-\overline{MP}$.

IN the ſame manner it will be found that if x, y, z, repreſent the reſpective Probabilites of three Lives continuing one Year, then the Probability of their not failing all three in one Year will be expreſſed by $x+y+z-xy-xz-yz+xyz$; which is ſufficient to ground this Concluſion, that an Annuity upon the longeſt of three Lives, is the Sum of the ſingle Lives, *minus* the Sum of the joint Lives, *plus* the three joint Lives: which has been expreſſed by me, by the Symbols $M+P+Q-\overline{MP}-\overline{MQ}-\overline{PQ}+\overline{MPQ}$.

FROM the foregoing Concluſions, it is eaſily perceived how the Value of the longeſt of any Number of Lives ought to be determined; *viz.* by the Sum of the Values of the ſingle Lives, *minus* the Sum of the Values of all the joint Lives taken two and two, *plus* the Sum of all the joint Lives taken three and three, *minus* the Sum of all the joint Lives taken four and four, and ſo on by alternate Additions and Subtractions.

CHAPTER IV.

Containing the Demonſtrations of what has been ſaid concerning Reverſions, and the Value of one Life after one or more Lives.

1°. It plainly appears that the preſent Value of a Reverſion after one Life, is the Difference between the Perpetuity, and the Value of the Life in Poſſeſſion: Thus, if the Life in Poſſeſſion be worth 14 Years Purchaſe, and that I have the Reverſion after that Life, and have a mind to ſell it, I muſt have for it 11 Years purchaſe, which is the Difference between the Perpetuity 25, and 14 the Value of the Life, when Money is rated at 4 *per Cent*.

2°. It is as evident that the Reverſion after two, three, or more Lives, is the Difference between the Perpetuity, and the longeſt of all the Lives.

But the Value of a Life after one or more Lives not being ſo obvious, I think it is proper to inſiſt upon it more largely: let x therefore repreſent the Probability of the Expectant's Life continuing one Year in being, and y the Probability of the ſecond Life's continuing alſo one Year in being, and therefore $1-y$ is the Probability of that ſecond Life's failing in that Year; from which it follows, according to the Doctrine of Chances, that the Probability of the firſt Life's continuing one Year, and of the ſecond's failing in that Year, is $x \times \overline{1-y}$, or $x-xy$; which is a ſufficient foundation for drawing the following Concluſion, *viz.* that the Value of the firſt Life after the ſecond is the Value of that firſt Life *minus* the Value of the two joint Lives: which I have expreſſed by the Symbols $M-\overline{MP}$.

In the ſame manner, if x, y, z, repreſent the reſpective Probabilities of three Lives continuing one Year, then $x \times \overline{1-y} \times \overline{1-z}$, will repreſent the Probability of the firſt Life's continuing one Year, and of the other two Lives failing in that Year; but the foregoing Expreſſion is brought, by actual Multiplication, to its Equivalent $x-xy-xz+xyz$; from whence can be deduced by meer Inſpection the Rule given in our 20th *pag.* *viz.* that the preſent Value of the firſt Life's Expectation after the Failing of the other two, is $M-\overline{MP}-\overline{MQ}+\overline{MPQ}$.

CHAPTER V.

Containing the Demonſtration of what has been aſſerted in the Solution of the 10*th and* 26*th Problems.*

In the Solution of the 10th Problem, M''' denoting the preſent Value of an Annuity to continue ſo long as three Lives of the ſame Age ſubſiſt together, let us ſuppoſe that n denotes the Number of Years during which the Annuity will continue; then ſuppoſing r to expreſs the Rate of Intereſt, it is well known that the preſent Value of that Annuity will be $\frac{1-\frac{1}{r^n}}{r-1}$, wherefore we have the Equation

$M''' = \frac{1-\frac{1}{r^n}}{r-1}$, or making $r-1=d$,

$M''' = \frac{1-\frac{1}{r^n}}{d}$, from whence will be deduced $\frac{1}{r^n} = 1 - dM'''$, and conſequently $r^n = \frac{1}{1-dM'''}$. Now let us ſuppoſe that a Sum s is to be received to eternity at the equal Intervals of Time, denoted by n, and that we want to find the preſent Value of it; it is plain to thoſe who have made ſome Proficiency in Algebra, that $\frac{s}{r^n-1}$ is the preſent Value of it, let us therefore in the room of r^n ſubſtitute its Value found before, *viz.*

$\frac{1}{1-dM'''}$, and then r^n-1 will be found equal to $\frac{dM'''}{1-dM'''}$, and conſequently $\frac{ſ}{r^n-1}=\frac{1-dM'''}{dM'''}\times ſ$: which is the Aſſertion of my 35th *Page*.

Now it will be eaſy to find n; for let us ſuppoſe $\frac{1}{1-dM'''}=T$, then $r^n=T$, and therefore $n=\frac{\log. T}{\log. r}$.

The 26th Problem has ſome Affinity with the 10th; in the former it was required to know the preſent Value of a Sum ſ, payable at the Failing of any one of three equal Lives, but in the latter the three Lives are ſuppoſed unequal; but beſides, it is extended to two other Caſes, *viz.* to the preſent Value of a Sum ſ to be paid after the Failing of any two of the Lives, as alſo to the preſent Value of a Sum ſ to be paid after the Failing of the three Lives.

For in the firſt Caſe, let us imagine an Annuity to be paid as long as the three Lives are in being; or, which is the ſame thing, till one of the Lives fails; and let us ſuppoſe that R repreſents the Value of the three joint Lives; let us alſo ſuppoſe that n is the Number of Years after which this will happen, and that d is the Intereſt of 1 *l*, therefore $\frac{ſ}{r^n}$ is the preſent Value of the Sum $ſ$ to be then paid; but $R = \frac{1 - \frac{1}{r^n}}{d}$, therefore $\frac{1}{r^n} = 1 - dR$, and therefore $\frac{ſ}{r^n} = \overline{1 - dR} \times ſ$.

BUT the ſecond Caſe has ſomething more of Difficulty, and therefore I ſhall enlarge a little more upon it: let us imagine now that there is an Annuity to continue not only as long as the three equal Lives are in being, but as long as any two of the ſaid Lives are in being; now in order to find the preſent Value of the ſaid Annuity, let us ſuppoſe that x, y, z, repreſent the reſpective Probabilities of the ſaid Lives continuing one Year. Therefore,

1°. xyz repreſents the Probability of their all outliving the Year.

2°. $xy\times\overline{1-z}$, or $xy-xyz$ repreſents the Probability of the two firſt outliving the Year, and of the third failing in that Year.

3°. $xz\times\overline{1-y}$ or $xz-xyz$ repreſents the Probability of the firſt and third's outliving the Year, and of the ſecond's failing in that Year.

4°. $yx\times\overline{1-z}$, or $yz-xyz$ repreſents the Probability of the ſecond and third's outliving the Year, and of the firſt's failing in that Year.

THEN adding thoſe ſeveral Products together, their Sum will be found equal to $xy+xz+yz-3xyz$, which is an Indication that the preſent Value of an Annuity to continue as long as two of the ſaid Lives are in being is $\overline{MP}+\overline{MQ}+\overline{PQ}-3\overline{MPQ}$, which we may ſuppoſe $=T$.

LET us now compare this with an Annuity certain to continue n Years, the Rate of Intereſt being ſuppoſed $=r$, and $r-1=d$, then we ſhall have the Equation $\frac{1-\frac{1}{r^n}}{d} = T$, from whence we ſhall find $\frac{1}{r^n} = 1 - dT$, and conſequently $\frac{s}{r^n}$, which is the preſent Value of the Expectation required, is $= \overline{1 - dT} \times s$.

By the ſame Method of Proceſs, we may find the preſent Value of an Annuity to continue ſo long as any one of the three Lives in queſtion is ſubſiſting; for let x, y, z, repreſent the ſame things as before.

1°. xyz repreſents the Probability of the three Lives outliving the firſt Year.

2°. $xy + xz + yz - 3xyz$ repreſents the Probability of two of them outliving the Year, and of the third's failing in that Year.

3°. $x \times \overline{1-y} \times \overline{1-z}$, or $x - xy - xz + xyz$ repreſents the Probability of the firſt Life's outliving the Year, and of the other two failing in that Year.

4°. $y\times\overline{1-x}\times\overline{1-z}$, or $y-xy-zy+xyz$ repreſents the Probability of the ſecond Life's outliving the Year, and of the other two failing in that Year.

5°. $z\times\overline{1-x}\times\overline{1-y}$, or $z-xz-yz+xyz$ repreſents the Probability of the third Life's outliving the Year, and of the other two failing in that Year.

Now the Sum of all this is $x+y+z-xy-xz-yz+zyz$; which is an Indication that the Value of an Annuity to continue as long as any one of three Lives is in being ought to be expreſſed by $M+P+Q-\overline{MP}-\overline{MQ}-\overline{PQ}+\overline{MPQ}$: and this laſt Caſe may be look'd upon as a Confirmation of the Rule given in our 5th Problem.

CHAPTER VI.

Containing the Demonſtration of what has been ſaid concerning ſucceſſive Lives pag. 33. *and following.*

WHAT has been there ſaid amounts to this; The preſent Values of Annuities certain for any particular Number of Years being given, to find the preſent Value of an Annuity to continue as long as the Sum of thoſe Years.

LET us ſuppoſe that M repreſents the preſent Value of an Annuity to continue n Years, and that P repreſents the preſent Value of an Annuity to continue p Years; the firſt Queſtion is, how from theſe *Data* to find the preſent Value of an Annuity to continue $n+p$ Years, the Inveſtigation of which is as follows: let r be the Rate of Intereſt, and ſuppoſe $r-1$ which denotes the Intereſt of 1 *l.* $=d$; then,

1°. $M=\frac{1-\frac{1}{r^n}}{d}$, therefore $\frac{1}{r^n}=1-dM$;

and for the ſame Reaſon $\frac{1}{r^p} = 1 - dP$. Therefore $\frac{1}{r^{n+p}} = \overline{1 - dM} \times \overline{1 - dP} = 1 - dM - dP + ddMP$. Let now $ſ$ be ſuppoſed to be the Value of the Annuity which is to continue $n + p$ Years, then $\frac{1}{r^{n+p}} = 1 - dſ$. Therefore $1 - dſ = 1 - dM - dP + ddMP$; then ſubtracting Unity on both Sides, dividing all by d, and changing the Signs, we ſhall have $ſ = M + P - dMP$.

2°. BY the ſame Method of Proceſs, it will be eaſy to find that if M, P, Q, repreſent Annuities to continue for the reſpective Number of Years n, p, q, then the Value of an Annuity to continue $n + p + q$ Years will be $M + P + Q - dMP - dMQ - dPQ + ddMPQ$: the Continuation of which is obvious.

LET us now ſuppoſe that the Intervals n, p, q, are equal, then the Values M, P, Q are alſo equal; in which caſe, the foregoing Canon will be changed into this, $3M - 3dMM + d^2M^3$, or $\frac{3dM - 3ddMM + d^3M^3}{d}$:

but if this Numerator be ſubtracted from Unity, the Remainder will be $1-3dM+3ddMM-d^3M^3=\overline{1-dM}^3$; and ſubtracting this again from Unity, the original Numerator will be reſtored, and will be equivalent to $1-\overline{1-dM}^3$, and conſequently, if M repreſents the Value of an Annuity to continue a certain Number of Years, then $\frac{1-\overline{1-dM}^3}{d}$ will repreſent the Value of an Annuity to continue three times as long.

AND univerſally, if M ſtands for the Value of an Annuity to continue a certain Number of Years, then $\frac{1-\overline{1-dM}^n}{d}$ will repreſent the Value of an Annuity to continue n times as long.

AND if n were infinite, I ſay that $\overline{1-dM}^n$ would be $=0$; from whence the Value would be $=\frac{1}{d}$ or $\frac{1}{r-1}$, which repreſents the Value of the Perpetuity.

BUT that there may remain no ſcruple about what we have aſſerted above, that in the Caſe of n being infinite, $\overline{1-dM}^{n}$ would vaniſh; I prove it thus, $\frac{1}{d} \urcorner M$, therefore $1 \urcorner dM$, therefore $1-dM$ is a Fraction leſs than Unity: now it is well known that a Fraction leſs than Unity being raiſed to an infinite Power, is nothing, and was therefore ſafely neglected.

CHAPTER VII.

Containing the Demonſtration of what has been aſſerted in the Pages 85 *and* 86, *concerning half-yearly Payments; as alſo the Inveſtigation of ſome Theorems relating to that Subject.*

IT is well known that if an Annuity A is to continue n Years, the preſent Value of it is $\frac{A-\frac{A}{r^{n}}}{r-1}$; ſuppoſing r to repreſent the Rate of Intereſt: now to make a proper Application of this Theorem to half-yearly Payments, I look upon n as repreſenting indifferently the Number of Payments and the Number of Years; let us now ſuppoſe a half-yearly Rent B of the ſame preſent Value as the former, and to continue as long, then the Number of Payments in this Caſe will be $2n$, but the Rate of Intereſt, inſtead of being r, is now $r^{\frac{1}{2}}$, which being raiſed to the Power $2n$, will be r^n as before; for which Reaſon the preſent Value of the half-yearly Payments is $\dfrac{B-\frac{B}{r^n}}{r^{\frac{1}{2}}-1}$: but by Hypotheſis, the preſent Values of the yearly and half-yearly Payments are the ſame; therefore $\dfrac{A-\frac{A}{r^n}}{r-1} = \dfrac{B-\frac{B}{r^n}}{r^{\frac{1}{2}}-1}$, and dividing both ſides of the Equation by $1-\frac{1}{r^n}$, we ſhall have $\dfrac{A}{r-1} = \dfrac{B}{r^{\frac{1}{2}}-1}$, from whence will be deduced $B = \frac{r^{\frac{1}{2}} - 1}{r - 1} \times A$: and in the ſame manner, if the Payments were to be made quarterly, then B would be $= \frac{r^{\frac{1}{4}} - 1}{r - 1} \times A$; and ſo on.

But if we ſuppoſe that a Rent ſhall be paid half-yearly, and that it ſhall alſo be one half of what would be given for an annual Rent, and that the two Rents ſhall be of the ſame Duration; then the preſent Values of the yearly and half-yearly Rents will be different: for let M and P be the preſent Values of the yearly and half-yearly Rents, then $M = \frac{A - \frac{A}{r^n}}{r - 1}$, and $P = \frac{\frac{1}{2}A - \frac{\frac{1}{2}A}{r^n}}{r^{\frac{1}{2}} - 1}$, and dividing both Values by $A - \frac{A}{r^n}$, we ſhall have $M, P :: \frac{1}{r - 1}, \frac{\frac{1}{2}}{r^{\frac{1}{2}} - 1}$; and conſequently $P = \frac{\frac{1}{2} \times \overline{r - 1}}{r^{\frac{1}{2}} - 1} \times M$.

THE two laſt Problems bring to my Mind an Aſſertion which was maintained, about ſix Years ago, in a Pamphlet then publiſhed; which was that it would be of great Advantage to a Perſon who pays an Annuity, to diſcharge it by half-yearly Payments, each of one half the Annuity in Queſtion: the Reaſon of which was, that then the time of paying off the Principal would be conſiderably ſhortened. I had not the Curioſity to read the Author's Calculation, becauſe I thought it too long; ſince which Time I thought fit to examine the thing, and found that indeed the Time would be ſhortened, but not ſo conſiderably as the Author imagined: which to prove, I ſuppoſed a Principal of 2000 *l.* an Annuity of 100 *l.* and the Rate of Intereſt 1.04: in conſequence of which, I found that the Principal would be diſcharged in 41 Years; this being founded on the general Theorem $\frac{A-\frac{A}{r^n}}{r-1}=P$, in which *A* repreſents the Annuity, P the Principal, r the Rate of Intereſt, and n the Number of Years: now to apply this to the Caſe of half-yearly Payments, let us ſuppoſe that p denotes the Number of Years in which the Principal will be diſcharged; therefore $2p$ will be the Number of Payments, $\frac{1}{2}A$ the Annuity, and $r^{\frac{1}{2}}$ the Rate of Intereſt: which being reſpectively ſubſtituted in the Room of n, A, r, we ſhall have now $\frac{\frac{1}{2}A-\frac{\frac{1}{2}A}{r^{p}}}{r^{\frac{1}{2}}-1} = P$, but $r^{\frac{1}{2}} - 1 =$ 0.019804, which being ſuppoſed $= m$, we ſhall have $\frac{1}{2}A - \frac{\frac{1}{2}A}{r^{p}} = mP$, and $\frac{\frac{1}{2}A}{r^{p}} = \frac{1}{2}A - mP$, or $\frac{50}{r^{p}} = 10.392$; therefore $\frac{r^{p}}{50} = \frac{1}{10.392}$, or $r^{p} = \frac{50}{10.392}$, and $p\ log.\ r = log.\ 50 - log.\ 10.392 =$ 0.6822709, therefore $p = \frac{0.6822709}{log.\ r}$; again, $log.\ r = 0.0170333$, therefore $p = \frac{0.06822709}{0.0170333} = 40.05$: and therefore the Advantage of paying half-yearly would amount to no more than gaining one Year in 41.

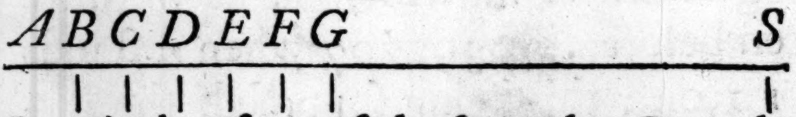
QUARTERLY Payments, or half-quarterly, nay even Payments made at every Inſtant of Time, would not much accelerate the Diſcharge of the Principal. Which to prove, let us reſume once more our general Theorem $\frac{A-\frac{A}{r^n}}{r-1}=P$; let us now imagine that the Number of Inſtants in the Year is $=t$, let us further ſuppoſe that s is the Number of Years in which the Principal will be diſcharged, then in the room of A, writing $\frac{1}{t}A$; in the room of r, writing $r^{\frac{1}{t}}$; and in the room of n, writing $s\,t$, we ſhall have $\frac{\frac{1}{t}A-\frac{\frac{1}{t}A}{r^s}}{r^{\frac{1}{s}}-1}=P$. But it is known, that if t repreſents an infinite Number, ſuch as is the Number of Inſtants in one Year, then $r^{\frac{1}{t}} - 1 = \frac{1}{t}\,log.\,r$, we have therefore $\frac{\frac{1}{t}A - \frac{\frac{1}{t}A}{r^s}}{\frac{1}{t}\,log.\,r} = P$, or $\frac{A - \frac{A}{r^s}}{log.\,r} = P$; let the Logarithm of r be ſuppoſed $= a$, therefore $A - \frac{A}{r^s} = aP$, and $\frac{A}{r^s} = A - aP$, and $r^s = \frac{A}{A - aP}$, which ſuppoſe $= \mathcal{Q}$, then $s = \frac{log.\,\mathcal{Q}}{log.\,r.}$: But it is to be noted, that a repreſents the hyperbolic Logarithm of r, which is, as we have ſeen before, 0,0392207 when r ſtands for 1,04; this being ſuppoſed, the Logarithm of $\mathcal{Q}$ will be found to be 0,6663794, which being divided by the Logarithm of r, *viz.* 0,0170333, the Quotient will be 39,1 Years; but in this laſt Operation the Logarithms of $\mathcal{Q}$ and r, may be taken out of a common Table.

CHAPTER VIII.

Containing the Demonſtration of what has been ſaid concerning the Probabilities of Survivorſhip.

WHAT I call Complement of Life having been defined before *pag.* 14. I ſhall proceed to make uſe of that Word as often as occaſion ſhall require.

HYPOTHESIS.



Let it be ſuppoſed that the Complement of Life AS being divided into an infinite Number of equal Parts repreſenting Moments, the Probabilities of living from A to B, from A to C, from A to D, *&c.* are reſpectively proportional to the ſeveral Complements SB, SC, SD, in ſo much that theſe Probabilities may reſpectively be repreſented by the Fractions $\frac{SB}{SA}$, $\frac{SC}{SA}$, $\frac{SD}{SA}$ *&c.* This Hypotheſis being admitted, the following Corollaries may be deduced from it.

COROLLARY I.

THE Probability of Life's failing in any Interval of Time AF is meaſured by the Fraction $\frac{FA}{SA}$.

COROLLARY II.

WHEN the Interval AF is once paſt, the Probability of Life's continuing from F to G is $\frac{SG}{SF}$, for at F, the Complement of Life is SF, and the Probability of its failing is $\frac{FG}{SF}$.

COROLLARY III.

THE Probability of Life's continuing from A to F, and then failing from F to G is $\frac{SF}{SA} \times \frac{FG}{SF} = \frac{FG}{SA}$.

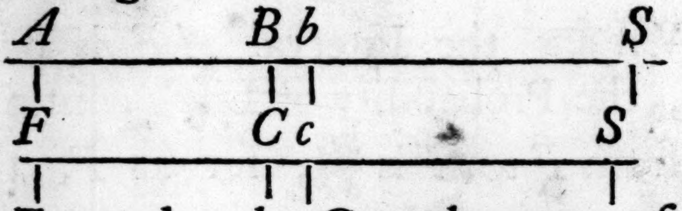
COROLLARY IV.

THE Probability of Life's failing in any two or more equal Intervals of

Time aſſigned between A and S are exactly the ſame, the Eſtimation being made at A conſidered as the preſent Time.

THESE things premiſed, it will not be difficult to ſolve the following Problem.

Two Lives being given, to find the Probability of one of them fixed upon, ſurviving the other.



FOR, let the Complements of the two Lives be reſpectively $AS = n$ and $FS = p$, upon which take the two Intervals AB, $FC = z$, as alſo the two Moments Bb, $Cc = \dot{z}$.

THE Probability of the firſt Life's continuing from A to B, or beyond it, is $\frac{n-z}{n}$; the Probability of the ſecond's continuing from F to C, and then failing in the Interval Cc, is by the third *Corollary* $\frac{\dot{z}}{p}$: therefore the Probability of the firſt Life's continuing during the time *AB* or beyond it, and of the ſecond's failing juſt at the end of that Time, is meaſured by $\frac{n-z}{n} \times \frac{\dot{z}}{p} = \frac{n\dot{z} - z\dot{z}}{np}$, whoſe Fluent $\frac{nz - \frac{1}{2}zz}{np}$ will expreſs the Probability of the firſt Life's continuing during any Interval of Time or beyond it, and of the ſecond's failing any time before or preciſely at the end of that Interval.

LET now p be written inſtead of z, and then the Probability of the firſt Life's ſurviving the ſecond, will be $\frac{np - \frac{1}{2}pp}{np} = 1 - \frac{\frac{1}{2}p}{n}$.

FROM the foregoing Concluſion we may immediately infer that the Probability of the ſecond Life's ſurviving the firſt is $\frac{\frac{1}{2}p}{n}$.

BY the ſame method of arguing, we may proceed to the finding the Probability of any one of any Number of given Lives ſurviving all the reſt, and thereby verifying what we have ſaid in *Pag.* 55 and 56.

FINIS.

E R R A T A.

Page		for	read
34,	*l.* laſt but one,	*no*	*to.*
35,	*l.* laſt but four,	$\frac{1-dM''}{dM''}\times s$,	$\frac{1-dM'''}{dM'''}\times s$.
37,	*l.* 6,	*dM''*	*dM'''*
43,	*l.* laſt,	Cube	Square.
59,	*l.* 12 and 17,	$\frac{qq}{6nq}$	$\frac{qq}{6np}$.
	l. 19,	4°	1°
63,	*l.* laſt,	9*np*	6*np*
64,	*l.* 2,	$\frac{p}{2p}$	$\frac{p}{2n}$
106,	*l.* 17,	*r—x*	*r—xy.*
107,	*l.* 1,	ſixth	eighteenth
108,	*l.* 4,	2*dMP*	2*dMPQ*
113,	*l.* 12,	20th	32d
117,	*l.* 15 and 19	3	2
120.	*l.* 4,	33	45

A few Miſtakes may likewiſe have happened in the Tables, as in Tab. VI. at the Age 68 *there ſtands the Value* 6.75, *inſtead of* 6.71. *But ſuch Miſtakes are of no Conſequence, being eaſily diſcovered and corrected by the Differences of the adjoining Numbers.*

not the [illegible]

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